A Complete Inspection Plan for Critical Components

by

Hazem Jawad Sadeg Al-Najjar

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

SYSTEM ENGINEERING

April, 1993

Order Number 1354119

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Al-Najjar, Hazem Jawad Sadeg, M.S.

King Fahd University of Petroleum and Minerals (Saudi Arabia), 1993

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This thesis, written by

Hazem Jawad Sadeg Al–Najjar

under the direction of his thesis committee, and approved by all the members, has been presented to and accepted by the Dean, College of Graduate Studies, in partial fulfillment of the requirement for the degree of

MASTER OF SCIENCE IN SYSTEMS ENGINEERING

Thesis Committee

Chairman (Dr. Salih O. Duffuaa)

Member (Dr. Abdul Raouf)

Member (Dr. Abdulrahman Shuaib)

Member (Dr. Jumah Al–Alwani)

Member (Dr. Mohammad Ben Daya)

Dr. Mohammad Al–Haboubi
Department Chairman

Dr Ala H. Al–Rabeh
Dean, College of Graduate Studies

Date: 17-4-93

كلية الدراسات العليا
COLLEGE OF GRADUATE STUDIES

To my dear father

and to the memory of my mother

whose support and prayers led to

this achievement.

Acknowledgements

All praise be to Allah, the Lord of the worlds, May peace and blessings be upon Mohammad the last of the messengers and his family. I thank Allah for His limitless help and guidance.

Acknowledgement is due to King Fahd University of Petroleum and Minerals for the generous help and support for this research.

I would like to express my deep and heart-felt gratitude and appreciation to my advisor Dr. S.O. Duffuaa, Associate Professor of Systems Engineering, for his guidance and patience throughout this work. His continuous support and encouragement can never be forgotten.

I would also like to thank the other committee members: Professor A. Raouf, Dr. A. Shuaib, Dr. J. Al-Alwani, and Dr. M. Ben Daya for their consistent support and cooperation.

The assistance of my friends can not go unacknowledged along with the hard-work of Mr. Mohammad Ayub Khan, the manuscript typesetter.

Finally, special thanks must be given to my family, especially my father, for their encouragement and moral support.

Table of Contents

Page

APPENDICES:

LIST OF TABLES

LIST OF FIGURES

Page

خلاصة الرسالة

إسم الطالب الكامل : حازم جواد صادق النجّار

عنوان الدراسة : خطة فحص كاملة للمُركّبات الحرجه .

التخصص : هندسة النظم والأساليب .

تاريخ الشهادة : أبريل ١٩٩٣م .

التحكم في الجودة موضوع يمكن تقسيمه الى فرعين هما : التحكم الإحصائي في عمليات الإنتاج والتحكم في جودة المنتج . وينقسم فرع التحكم في جودة المنتج الى خطط الفحص الكامل وخطط القبول بالعينات . وقد أصبحت خطط الفحص الكامل ذات أهمية بالغة في مجال التحكم في الجوده ، ويرجع ذلك الى التطور الكبير والهائل في نظم التصنيع الحديثه والتي جعلت خطط الفحص الكامل غير مكلفه وذات إعتماديه عاليه .

وتطور هذه الأطروحه خطة جديدة للفحص الكامل والمتكرر للمُركّبات الحرجه ذات الخواص المتعدده . وتم إستنباط النماذج الرياضية والتحاليل الخاصة بهذه الخطه . ولتطبيق النماذج الرياضيه الجديده تم تطوير برامج لحلّها ومقارنتها بالنماذج المنشوره من قبل ، وتمت المقارنه بإستخدام مسائل فحص أُنتجت عشوائياً . كما تم إدخال عملية نقل ومباشرة المواد في خطط الفحص ونماذجها الرياضية .

وقد أوضحت النتائج تميّز النموذج العام والذي يرمز اليه بـ نموذج -٢ في الأطروحه ، وذلك لقدرته على تخفيض تكلفة الفحص الكليه وتحسين الجوده في بعض الأحيان مقارنة بالنماذج المنشوره . وقد أعطى النموذج العام نتائج أفضل بنسبة تتراوح بين ٧٥٪ و ٨٠٪ من مجموع المسائل التي تم حلّها . كما تراوحت نسبة خفض تكلفة الفحص الكلية بين ١٪ و ٢٠٪ . وقد أدّت إضافة تكلفة نقل ومباشرة المواد الى عملية الفحص الى تحسين أداء الخطة المقترحة مقارنة بالخطط المنشوره ، وكان ذلك متوقعاً بسبب قلّة تكلفة نقل ومباشرة المواد في الخطة المطوره .

درجة الماجستير في العلوم

جامعة الملك فهد للبترول والمعادن

الظهران - المملكة العربية السعودية

THESIS ABSTRACT

FULL NAME OF STUDENT : HAZEM JAWAD SADEG AL-NAJJAR

TITLE OF STUDY : A COMPLETE INSPECTION PLAN FOR CRITICAL COMPONENTS

MAJOR FIELD : SYSTEMS ENGINEERING

DATE OF DEGREE : APRIL, 1993

Quality control can be divided into statistical process control and product control. The product control area can also be divided into complete inspection plans and acceptance sampling plans. Complete inspection plans are becoming increasingly important in the area of quality control due to the growth in modern manufacturing systems that make complete inspection inexpensive and reliable.

This thesis proposes a new complete inspection plan for multicharacteristic critical components. Optimization models which depict the plan are developed with the necessary algorithms and software. Comparisons with the existing models in the literature are carried out using randomly generated inspection problems. Efforts to explicitly incorporate material handling cost in the inspection process are made.

The general model in the thesis referred to as model (2) performed better in terms of expected total cost of inspection than the models in the literature, in 75–80 percent of the randomly-generated problems. The savings in total expected costs range from 1–20 percent. Incorporating material handling cost improved the performance of the proposed plan compared to the ones in the literature. This was expected due to less material handling involvements in the proposed plan.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
Dhahran, Saudi Arabia

April, 1993

CHAPTER 1

INTRODUCTION

1.1 Historical Review (Statistical Quality Control in Practice)

Quality control is as old as industry itself. From the time man began to manufacture, there has been an interest in quality of output. In more modern times, factory inspection and research, pure food and drug acts, activities of professional societies, all have sought for years to assure the quality of output. Quality control has thus had a long history.

On the other hand, statistical quality control is new. The science of statistics itself goes back only two to three centuries, and its greatest development has been in the last 70 years. Early applications were made in astronomy and physics and in the biological and social science, but it was not until the 1920s that statistical theory began to be applied effectively to quality control. In that decade, a factor in the birth of statistical quality control was the development, in immediately preceding years, of an exact theory of sampling.

Statistical quality control can be divided into two main areas:

1. Statistical process control.

2. Product control.

The product control area can also be divided into

(a) Complete inspection plans.

(b) Acceptance sampling plans.

Acceptance sampling is likely to be used under the following conditions:

(i) *When the cost of inspection is high and the loss arising from passing a defective unit is not great.* It is possible in some cases that no inspection at all will be the cheapest plan.

(ii) *When 100 percent inspection is fatiguing and a carefully worked-out sampling plan will produce as good or better results.* As noted above, 100 percent inspection may not mean 100 percent perfect quality, and the percentage of defective items passes may be higher than under a scientifically designed sampling plan.

(iii) *When inspection is destructive.* In this case, sampling must be employed.

The complete inspection or 100 percent inspection was thought to be impossible, but the modern means of science and technology proved that this claim is not true. Complete inspection nowadays is possible and applicable. Therefore, complete inspection plans can be used, especially in situations where the loss arising from passing defective units is high or catastrophic.

The emphasis of this thesis work is on a branch of inspection plans. commonly known as *complete repeat inspection plans.* Complete inspection plans are becoming increasingly important in the area of quality control due to the developments in modern manufacturing systems that make complete inspection relatively inexpensive and reliable.

1.2 Complete Inspection Plans

Inspection can be defined as the function of comparing or determining the conformance of products to established specifications. A short definition could be "a procedure for determining faulty components."

The output of production systems invariably contains some defective items (units) caused by workers, machines or a combination of both. The inspection process is the basic source of information on these errors. Inspection tasks may be classified into three basic categories: tasks involving visual scanning, tasks involving measurements, and tasks involving monitoring of a process.

1.3 Some Inspection Terminology

1.3.1 Accuracy of Inspection

The accuracy of inspection process is the key element for better product quality.

This accuracy is influenced by:

1. Inspector–related factors (Examples include: visual acuity, age, experience, sex, intelligence, social factors, psychological factors, number of inspectors, eye movement, sleep deprivation, personality and level of training).

2. Task–related factors (Examples include: paced vs unpaced, defect conspicuity, task complexity, density of items, rate of tasks, importance of the characteristic being inspected, design of work place, time of the day, number of faults occurring simultaneously, method of inspection, inspection rate, display characteristics and task perception).

3. Environmental and organizational factors (Examples include: illumination, noise, temperature, humidity, motivation, incentives. music while working, management standards, job rotation and feed–back and feed–forward).

Because of these factors the inspector usually makes two types of errors: type I and type II errors.

1.3.2 Type I Error and Type II Error

Type I error is defined as: classifying a nondefective characteristic as defective. Type II error is defined as: classifying a defective characteristic as nondefective (*i.e.* failing to report a defect).

Type I error and type II error can be summarized in the following decision matrix:

	Decision Based on Inspection	
	Accept	Reject
Conforming	Correct decision	Wrong decision Type I error
Non-conforming	Wrong decision Type II error	Correct decision

1.3.3 Inspector Accuracy

It is the degree to which the inspector makes correct decisions on product quality. It is measured by the probability of making type I and type II errors. Figure 1.1, obtained from (19), shows the effect of inspection error on the production processes.

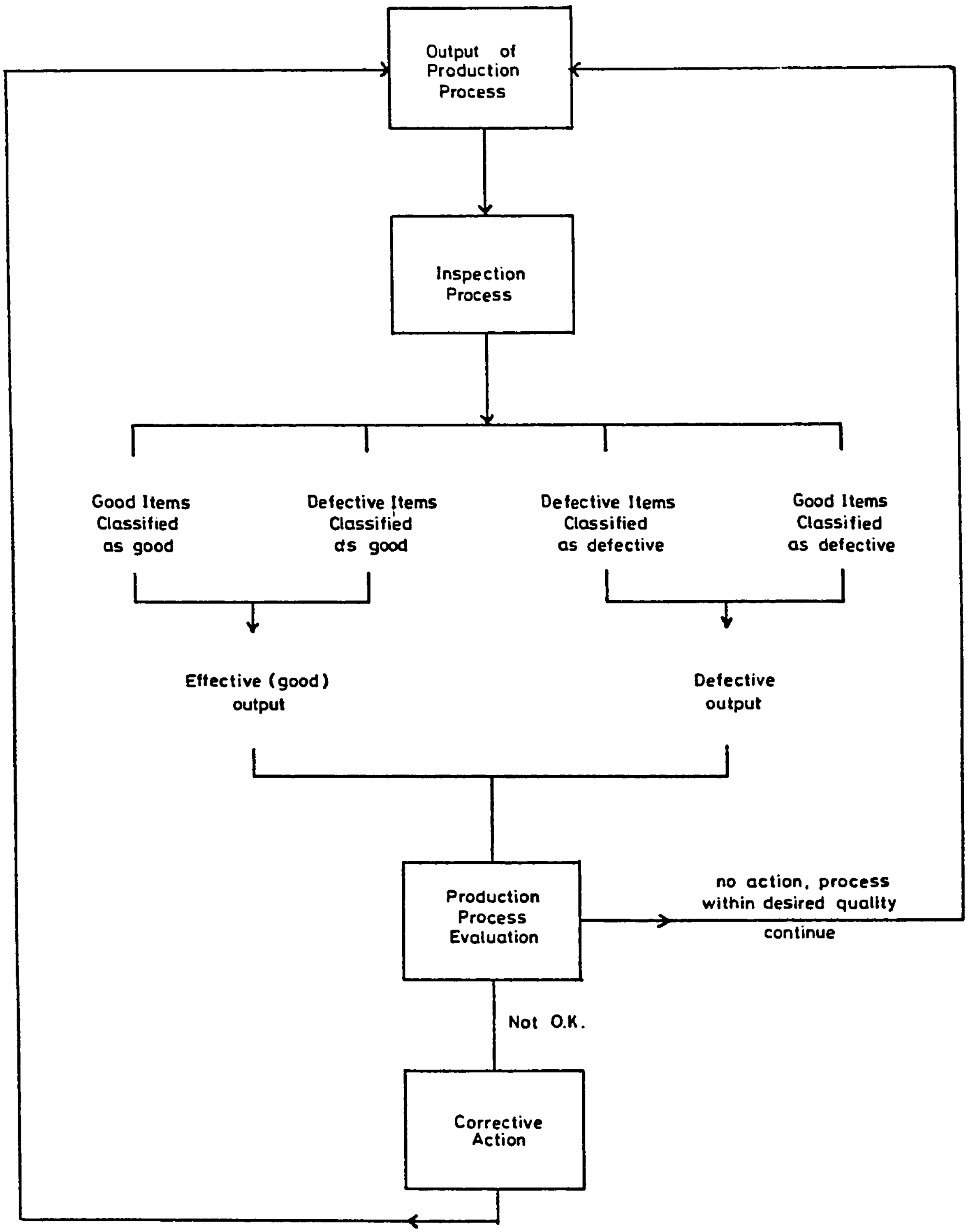

Fig.1.1 Block diagram illustrates the effect of inspection error on production processes.

1.3.4 Critical Components

A critical component is a component which, upon failure, may endanger human life or cause high cost. Such components usually have many characteristics which require inspections. Critical components could be parts of:

1. A nuclear reactor
2. An air–craft
3. A gas ignition system
4. A space shuttle

1.3.5 Repeat Inspection

A component failure is due to a defect in one or more of its characteristics. In order to guard against such failures, a common practice in industry is to institute repeat inspection (*i.e.*, the use of inspection redundancy). Since there is always a possibility of type I and type II errors, this repeat inspection could overcome the unperfectness of the inspection process.

Based on the literature review, it was found out that all the models on repeat inspection were based on a single inspection plan which was originally developed by Raouf *et al* (1983). This plan given in Figure 2.1 requires the number of inspections for all characteristics to be equal. Later, Duffuaa and Raouf (1989) and Duffuaa and Nadeem (1992) extended this model for components whose characteristics' defective rates are statistically dependent. In addition, the material handling cost has not been incorporated explicitly in the inspection models. Usually, different characteristics of a component have different defective rates and cost of inspection, and may require unequal number of repeat inspections. Therefore, a need to develop a plan which allows variable number of inspections for different characteristics is realized and will extend the models in the literature. Furthermore, incorporating material handling cost will make the models more realistic.

1.4 Thesis Objectives

The overall objective of this thesis is to generalize and extend the inspection plans and the models in the area of repeat inspection. Specifically, the objectives of this thesis are:

1. To propose a new inspection plan which allows variable number of inspections for different characteristics.
2. To develop optimization models which depict the new plan.
3. To develop a solution procedure and implement it for the models in (2).
4. To explicitly incorporate material handling costs in the inspection plans developed and the ones in the literature.
5. To compare the results of the models for the new plan developed in (2) and

(4) with those in the literature.

1.5 Plan of the Proposed Work

The proposed research work in the thesis starts with the development of a new inspection plan which allows for variable number of inspections for different characteristics. Then a model which depicts the new plan is developed, with the necessary computational procedures.

The model is developed in two stages. In the first stage, it is assumed that all characteristics will be inspected the same number of times: then, this assumption is relaxed to allow variable number of inspections and a general model is obtained. Comparisons among the models developed in the thesis and the models in the literature are carried out to identify situations where the new models reduce total cost of inspection.

Later, the cost of material handling is explicitly incorporated in the inspection process in order to generalize and integrate the inspection process in the manufacturing system.

1.6 Thesis Organization

This thesis is presented in seven chapters. Following the introduction, chapter 2 presents some background on repeat inspection, extensive review of the relevant literature and some fundamental models in the repeat inspection area.

Chapter 3 offers a new repeat inspection plan together with the optimization model which depicts the plan. The plane decentralizes the inspection process and allows for variable number of inspections for different characteristics. Computational procedures and comparisons with the models reported in the literature are also carried out in this chapter.

In chapter 4, the model in chapter 3 is modified to allow variable number of inspections for different characteristics. An algorithm to obtain the optimal number of repeat inspections is developed. Two examples are presented to demonstrate the results of the model.

The results of the detailed comparisons of the models developed in the thesis with those in the literature are given in chapter 5. The comparisons were carried out using randomly generated inspection problems.

The cost of material handling, incorporated explicitly in the inspection models developed in chapters 3 and 4, will be the subject of chapter 6. Chapter 7 provides the conclusions and directions for future research.

CHAPTER 2

INSPECTION MODELS AND LITERATURE REVIEW

2.1 Introduction

Inspection as defined earlier is a procedure for checking whether a product quality conforms with the required attributes and may result in detecting faulty components. In production systems, inspection for acceptance purposes is carried out at many stages. This includes inspection of incoming material, in process inspection during the manufacturing operation and finished product inspection. Competitive forces and sometimes catastrophic failures have resulted in tight quality control of product components. This is especially true for firms producing high technology products. Components whose failures result in catastrophic or serious hazard or which result in a very high cost are termed critical components. Such components have many characteristics which must conform to tight product specifications. A component is classified as defective and hence rejected or sent for rework, if one or more of its characteristics do not meet quality specifications. Examples of such components could be a part of a gas ignition system, a space shuttle, an aircraft or a nuclear reactor.

To guard against high cost or serious hazard, repeat inspection is usually instituted to reduce inspection errors. An inspector may commit type I error (*i.e.* classifying a nondefective component as defective) or type II error (*i.e.* classifying a defective component as nondefective). The initial inspection plan for critical components developed by Raouf *et al* (1983) involves sequential inspection of all characteristics of the component. The completion of inspection of all characteristics constitutes a cycle of inspection. In order to assure product quality, more than one cycle of inspection may be required. Accepted components from the first cycle if needed go to the next cycle of inspection. To minimize the total cost this may be repeated n cycles. The total cost of the inspection plan for critical components includes the cost of false acceptance of a defective component C_a, the cost due to false rejection C_r, and cost of inspection of each characteristic i, C_i.

The purpose of an optimal inspection plan is to determine the required number of cycles of inspection and the order in which characteristics are sequenced for inspection in order to minimize the total inspection cost. Although the models for optimizing repeat inspections were developed for a single plan which requires the number of inspections for all characteristics to be equal, situations may arise where the product characteristics may require unequal number of inspections.

In light of these above, the main purpose of this thesis is to extend the repeat inspection plans and optimization models developed by Raouf *et al* (1983) by proposing a new inspection plan which allows variable inspections for different characteristics. In addition, computational procedures and sequencing rules will be outlined and implemented to obtain the optimal number of repeat inspections.

Therefore, this chapter is organized in the following manner: section 2 presents the literature review; section 3 presents models in repeat multicharacteritsic inspection reported in the literature, while conclusions are contained in section 4.

2.2 Literature Review

Harris (1968) was the first to examine the effect of incoming quality on inspection accuracy. He concluded that inspection accuracy decreases with reduction in defect rate. Later, Raouf and Elfeituri (1983) conducted a study and listed the factors that may affect the inspector accuracy and studied the effect on inspector accuracy by varying incoming quality, task complexity and inspection rate. They came up with a conclusion that the probability of making type II error seems to be a more realistic criterion for measuring inspector accuracy than the probability of type I error. Ayoub *et al* (1970) defined mean inspection error to be the average number of defective items classified as good items by the inspector. They showed that this error has significant effect on cost measures of sampling plans and presented formulas for Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) for a single sampling plan under inspection error. Later, Collines *et al* (1972) relaxed the assumption of perfect inspection of replacement and allowed defective replacement in the formula for ATI. Garcia–Diaz *et al* (1983) presented a dynamic programming (DP) model for repeat 100 percent inspection. Garica–Diaz model was further analyzed by El–Magharby (1986) who presented an alternative condition for the applicability of the DP model.

Raouf *et al* (1981) developed the initial model for determining the optimal number of repeat inspections for complex units to minimize the total cost. The model considers a component with several critical characteristics to be inspected. Failure to meet the quality requirements of any characteristic results in the rejection of the component. The number of critical defects per component is assumed to have Poisson distribution. The solution obtained minimizes the total cost incurred in all aspects of an inspection plan and in this sense the solution is a global minimum.

Raouf *et al* (1983) revised the initial model for determining the optimal number of repeat inspection for multicharacteristic components to minimize the total cost per accepted component due to type I error, type II error and cost of inspection. A sequencing rule is proposed for minimizing the cost of inspection within each inspection cycle. A proof of this rule was later provided by Duffuaa and Raouf (1990).

Duffuaa and Raouf (1987) extended the model for statistically dependent multicharacteristic components. They developed a procedure under the assumption that the probability of a characteristic being defective depends upon the probability of other characteristics being defective, for finding the optimal sequence of inspection in each inspection cycle. Duffuaa and Raouf (1989) developed three mathematical optimization models for multicharacteristic repeat inspections. The first model (cost minimization model) minimizes the total cost due to type I error, type II error and repeat inspection to determine the optimal number of repeat inspection. The second model (probability minimizing model) reduces the probability of accepting a defective component. The third model (the satisfying model) determines a satisfying solution by specifying an upper limit for total inspection cost and the probability of accepting a defective component. Lee (1988) simplified Raouf *et al* model and obtained simple optimality conditions for the model.

Chandra and Susan (1980) presented a method of optimizing total inspection cost assuming that measurement error is random variable $\sim N(0, \sigma^2)$ while the true value of the product $\sim (u, \sigma^2)$. They suggested that in order to obtain the best estimator of the measurement of interest an optimum number of replicates should be used. Bennet *et al* (1974) investigated the effect of errors on a single sampling plan with known incoming quality. Tang (1987) investigated the economic and statistical impacts of inspector errors on complete inspection plans. Tang borrowed Taguchi's concept of quadratic loss function to determine the expected quality cost per item after inspection. Two models were developed for complete inspection with inspection error considerations, and compared to the model without inspection error consideration. Inspection error is shown to have a significant effect on the inspection limits, proportion of reworked items, total cost and other plan characteristics. However, these effects become insignificant when inspection precisions are high.

Duffuaa (1992) examined the effect of inspector error on the expected total cost of a complete repeat inspection plan for critical components. Also, Duffuaa and Nadeem (1992) developed an extension of the model proposed in Raouf *et al* (1983) for components whose characteristics' defective rates are statistically dependent.

Yumei and Tang (1990) studied the economic design of multicharacteristic models for Three–Class Screening Procedure. Tang and Helmut (1987) studied the effect of inspection error on a complete inspection plan. The effects of inspection error on Average Outgoing Quality, the operation of a C–chart and performance measures of multistage sampling plan is given in Case *et al*, (1975), Dorris (1977) and Maghsoodloo (1987) respectively. Wang (1989) defined, studied and investigated the inspection effort allocation problem in sequential multistage production systems. He stated that the concept of a quality transfer factor makes it possible to treat inspection stations and production stations equally.

In the literature, all the developed models on repeat inspection plans were based on a single inspection plan which was originally developed by Raouf *et al* (1983). This plan requires the number of inspections for all characteristics to be equal. Later Duffuaa and Raouf (1987) and Duffuaa and Nadeem (1992) extended this model for statistically dependent multicharacteristic components. In addition, the material handling cost has not been incorporated explicitly in the inspection models. Therefore a need to develop a plan which allows variable inspections for different characteristics is realized and will extend the models in the literature. Incorporation of the material handling cost will also make the models more realistic.

2.3 Models for Repeat Multicharacteristic Inspection

It was mentioned earlier in this chapter that many models were developed in the area of complete repeat inspection of critical components. These models were developed to determine the optimal number of repeat inspections for multicharacteristic critical components which minimizes the total cost per accepted component due to the cost of false acceptance, cost of false rejection and repeat inspection cost.

It is assumed that each component consists of N characteristics. Each characteristic has a probability P_i of being defective. Inspectors usually commit type I, P_{1i} and type II, P_{2i} errors. Inspection of each characteristic costs C_i, involves also cost of false rejection C_r and cost of false acceptance C_a. These data are usually assumed to be available.

Raouf *et al* (1983) developed the initial cost minimization model for multicharacteristic component inspection. This model will be used later for comparison with the models in the thesis. Therefore, it is felt appropriate to present this model, and is given in the next subsections.

2.3.1 Model Description

The model is developed for components containing several characteristics for inspection with known incoming quality P_i. A component is classified as nondefective only if all the characteristics meet the quality specifications. The probabilities of type–I error. P_{1i} and type–II error P_{2i} are assumed to be known. Three different types of costs are considered: (i) cost due to false rejection of a nondefective component, C_r. (ii) cost due to false acceptance of a defective component, C_a, and, finally (iii) cost of inspection, C_i. The estimates for C_r, C_a, and C_i are assumed to be available in the industry.

The inspection plan shown in Figure 2.1 is as follows: an inspector inspects one particular characteristic for each component entering the inspection process. All the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the component is finally accepted. Here n is the optimal number of inspections necessary to minimize the total cost per accepted component. Finally, the accepted components will be those which are accepted in the n-th cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, ..., n-th cycles. In order to model this inspection plan, the following notation is employed.

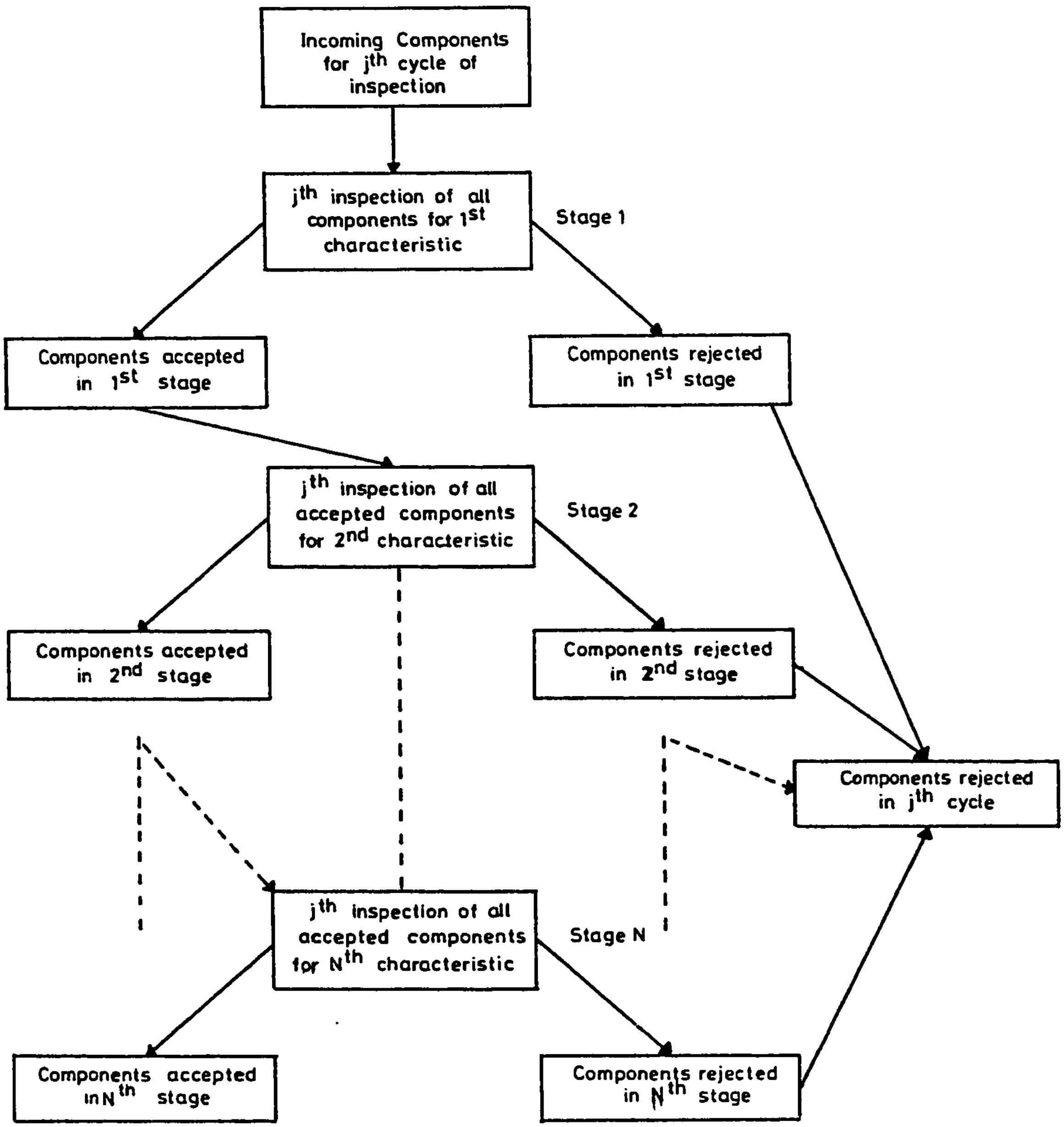

Fig. 2.1 Inspection plan for j cycle. $j = 1, 2, \ldots, n$.

Notation

M_j	Number of components entering the j-th cycle of inspection
N	Number of characteristics in each component to be inspected
P_i	Probability of i-th characteristic in the sequence of inspection being defective entering the inspection
PG	Probability of a component being nondefective entering the inspection
PGC	Probability of a component being defective entering the inspection, the complement of PG
P_{1i}	Probability of classifying the i-th nondefective characteristic in the sequence of inspection as defective (Type–I error)
P_{2i}	Probability of classifying the i-th defective characteristic in the sequence of inspection as nondefective (Type–II error)
$P_i(j)$	Probability of i-th characteristic in the sequence of inspection being defective entering the j-th cycle
$PG(j)$	Probability of a component being nondefective entering the j-th cycle
$PGC(j)$	Probability of a component being defective entering the j-th cycle, complement of $PG(j)$
$M_{i,j}$	Number of components entering the i-th stage of inspection in the j-th cycle
$PG_{i,j}$	Probability of a component being nondefective in the i-th stage of the j-th cycle

$FR_{i,j}$ Number of falsely rejected components in the i-th stage of the j-th cycle

$FA_{i,j}$ Number of falsely accepted components in the i-th stage of the j-th cycle

$CA_{i,j}$ Number of correctly accepted components in the i-th stage of the j-th cycle

$R_{i,j}$ Rate of rejection of components due to i-th characteristics in the sequence of inspection in the j-th cycle

$A(j)$ Number of accepted components in the j-th cycle

$CFR(j)$ Cost of false rejection in the j-th cycle

$CFA(j)$ Cost of false acceptance in the j-th cycle

$CI(j)$ Cost of inspection in the j-th cycle

$TCFR$ Total cost of false rejection

$TCFA$ Total cost of false acceptance

TCI Total cost of inspection

TA Total number of accepted components

$E(tc)|_j$ Expected total cost per accepted component after j cycles of inspection

$E(\ )$ Expected value of the argument inside the parentheses

2.3.2 Basic Relationships in the Model

The probability of the i-th characteristics being defective will vary from cycle to cycle. The relationship between $P_i(j)$ and P_i is given below.

$$P_i(1) = P_i \tag{2.1}$$

Using Baye's theorem

$$P_i(2) = P_i P_{2i}/[P_i P_{2i} + (1 - P_i)(1 - P_{1i})] \tag{2.2}$$

Similarly

$$P_i(3) = P_i(2)P_{2i}/[P_i(2)P_{2i} + (1 - P_i(2))(1 - P_{1i})] \tag{2.3}$$

and from the symmetry of expressions (2.2) and (2.3) we get

$$P_i(j) = P_i(j-1)P_{2i}/[P_i(j-1)P_{2i} + (1 - P_i(j-1)(1 - P_{1i})] \tag{2.4}$$

The probability of a characteristic being defective changes in each cycle; hence the probability of a component being nondefective also changes. It is given below:

$$PG = \prod_{i=1}^{N}(1 - P_i) \tag{2.5}$$

The probability of a component being defective is

$$PGC = 1 - PG \tag{2.6}$$

Clearly,

$$PG(1) = PG = \prod_{i=1}^{N}(1 - P_i(1)) \tag{2.7}$$

The probability of a component being nondefective entering the j-th cycle is

$$PG(j) = \prod_{i=1}^{N}(1 - P_i(j)) \tag{2.8}$$

The probability of a component being defective entering the j-th cycle is

$$PGC(j) = 1 - PG(j) \tag{2.9}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost of false acceptance of all the defective components, *i.e.*,

$$E(tc)|_{j=0} = C_a(1 - PG) \tag{2.10}$$

The expected total cost per accepted component, after n cycles of inspection. can be written as

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{2.11}$$

where $TCFR. TCFA, TCI$, and TA are as defined earlier.

2.3.3 Cost Minimization Model

The objective of this model is to determine the optimal inspection plan for multicharacteristic components. The model minimizes the total cost per accepted component resulting from type I errors, type II errors, and cost of inspection. Given the basic relationships in the previous section, a mathematical expression for expected total cost per accepted component will be obtained. Our objective is to minimize this cost subject to the relationships governing this situation.

In order to derive the cost of inspection after n cycles of inspections, analysis of cycle 1 of inspection is necessary. All the components entering the cycle 1 go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. This is the first stage of inspection.

Stage 1 in cycle 1: Number of components entering this stage is

$$M_{1,1} = M_1 \tag{2.12}$$

The probability of a component being nondefective is

$$PG_{1,1} = PG \tag{2.13}$$

E (number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1} PG_{1,1} P_{11} \\ &= M_1 PG P_{11} \end{aligned} \tag{2.14}$$

E (number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1}[P_1 P_{21} + (1 - PG_{1,1} - P_1)(1 - P_{11})] \\ &= M[P_1 P_{21} + (1 - PG - P_1)(1 - P_{11})] \end{aligned} \tag{2.15}$$

E (number of correctly accepted components) is

$$\begin{aligned} CA_{1,1} &= M_{1,1} PG_{1,1}(1 - P_{11}) \\ &= MPG(1 - P_{11}) \end{aligned} \tag{2.16}$$

All accepted components in this stage go to the second inspector who inspects the second characteristic of each component in order to classify it as defective or nondefective.

Stage 2 of the first cycle

$$\begin{aligned}M_{2,1} &= FA_{1,1} + CA_{1,1} \\ &= M_1[P_1P_{21} + (1-P_1)(1-P_{11})]\end{aligned} \tag{2.17}$$

$$\begin{aligned}PG_{2,1} &= CA_{1,1}/M_{2,1} \\ &= PG(1-P_{11})/[P_1P_{21} + (1-P_1)(1-P_{11})]\end{aligned} \tag{2.18}$$

$$\begin{aligned}FR_{2,1} &= M_{2,1}PG_{2,1}P_{12} \\ &= M_1PG(1-P_{11})P_{12}\end{aligned} \tag{2.19}$$

$$\begin{aligned}FA_{2,1} &= M_{2,1}[P_2P_{22} + (1-PG_{2,1}-P_2)(1-P_{12})] \\ &= M_1[P_2P_{22} + (1-PG_{2,1}-P_2)(1-P_{12})]\end{aligned} \tag{2.20}$$

$$\begin{aligned}CA_{2,1} &= M_{2,1}PG_{2,1}(1-P_{12}) \\ &= M_1PG\prod_{i=1}^{2}(1-P_{1i})\end{aligned} \tag{2.21}$$

By symmetry, we can obtain stage N of the first cycle

$$M_{N,1} = M_1\prod_{i=1}^{N-1}[P_iP_{2i} + (1-P_i)(1-P_{1i})] \tag{2.22}$$

$$PG_{N,1} = PG\prod_{i=1}^{N-1}[(1-P_{1i})/(P_iP_{2i} + (1-P_i)(1-P_{1i})] \tag{2.23}$$

$$FR_{N,1} = M_1PG\prod_{i=1}^{N-1}(1-P_{1i})P_{1N} \tag{2.24}$$

$$\begin{aligned}FA_{N,1} = M_1 &\prod_{i=1}^{N-1}[P_1P_{2i} + (1-P_i)(1-P_{1i})] \\ &\times[P_NP_{2N} + (1-PG_{N,1}-P_N)(1-P_{1N})]\end{aligned} \tag{2.25}$$

$$CA_{N,1} \doteq M_1PG\prod_{i=1}^{N}(1-P_{1i}) \tag{2.26}$$

This completes one cycle of inspection, and the result of this cycle is described by the following equations. Number of accepted components after completing the first cycle is,

$$A(1) = FA_{N,1} + CA_{N,1} \tag{2.27}$$

Cost of false rejection is

$$CFR(1) = C_r \sum_{i=1}^{N} (FR_{i,1}) \tag{2.28}$$

Cost of false acceptance is

$$CFA(1) = C_a(FA_{N,1}) \tag{2.29}$$

Cost of inspection is

$$CI(1) = \sum_{i=1}^{N} C_i M_{i,1} \tag{2.30}$$

E (total cost per accepted components after one cycle of inspection is)

$$E(tc)|_{j=1} = [CFR(1) + CFA(1) + CI(1)]/A(1) \tag{2.31}$$

where $CFR(1)$, $CFA(1)$, $CI(1)$, and $A(1)$ are given by equations (2.28), (2.29), (2.30) and (2.27), respectively.

Before proceeding to the second cycle, it was shown in Raouf *et al* (1983), that the manner in which characteristics are ordered for inspection affects the cost of inspection; in Duffuaa and Raouf (1990), a rule is given and proved to be optimal for minimizing the cost of inspection within each inspection cycle. In minimizing the cost, this rule should be applied in each cycle. The rule says, "at inspection cycle j, compute the ratio $C_i/R_{i,j}$ for all i; then, for each component, first inspect the characteristic with the least ratio, and lastly, inspect the one with the highest ratio." This value ensures that $CI(j)$ is minimized within each cycle.

From the analysis of cycle 1, it can easily be seen that after this cycle, we can compute the new values of $P_i(2)$, $PG(2)$, M_2 and proceed in the same manner as in the first cycle to compute the cost of false rejection, cost of false acceptance and cost of inspection in this cycle. Hence, by symmetry, we can obtain the results of the n-th cycle.

$$A(n) = FA_{N,n} + CA_{N,n} \tag{2.32}$$

$$CFR(n) = C_r \sum_{i=1}^{N} (FR_{i,N}) \tag{2.33}$$

$$CFA(n) = C_a (FA_{N,n}) \tag{2.34}$$

$$CI(n) = \sum_{i=1}^{N} C_i M_{i,n} \tag{2.35}$$

The ratio used to determine the optimal ordering of characteristics in the n-th cycle is, $C_i/R_{i,n} \quad i = 1, 2, \ldots, N$ where

$$R_{i,n} = P_i(n)(1 - P_{2i}) + (1 - P_i(n))P_{1i} \tag{2.36}$$

After n cycles of inspection we must determine the total cost of inspection per accepted component, which consists of: total cost of false rejection $TCFR$, total cost of false acceptance $TCFA$, and total cost of inspection.

$$TCFR = \sum_{j=1}^{n} [CFR(j)] \tag{2.37}$$

$$TCFA = CFA(n) \tag{2.38}$$

$$TCI = \sum_{j=1}^{n}[CI(j)] \tag{2.39}$$

Total accepted components

$$TA = A(n) \tag{2.40}$$

The above equations (2.1) through (2.40) provide the basic relationship for the model; the purpose is to find the value of n which minimizes the expected total cost per accepted component. The above model can be stated as

$$\text{Min } E(tc)|_{j=n} \tag{2.41}$$

The following is an algorithm for finding a local optimal n.

2.3.4 Algorithm

Step 1 Determine the PG and $E(tc)|_{j=0}$ from equations (2.5) and (2.10); respectively, set $j = 1$.

Step 2 Compute $P_i(j), PG(j), PG_{N,j}, M_j$ and $C_i/R_{i,j}$ for $i = 1, 2, \ldots, N$ using equations (2.4), (2.8), (2.23), (2.27), (2.36), respectively. Arrange the ratios $C_i/R_{i,j}$ ($i = 1, 2, \ldots, N$) in order of decreasing magnitude. This is the optimal sequence for the j-th cycle and has been shown in (Duffuaa and Raouf, 1990).

Step 3 Rearrange the probabilities P_i, P_{1i}, P_{2i}, and the inspection cost C_i according to the optimal sequence obtained in step 2.

Step 4 Compute $A(j), CFR(j), CFA(j)$, and $CI(j)$ using equations (2.32), (2.33), (2.34), and (2.35) respectively.

Step 5 Compute $TCFR$, $TCFA$, TCI, and TA from equations (2.37), (2.38), (2.39), and (2.40) respectively.

Step 6 Compute $E(tc)|_j$, using equation (2.11).

Step 7 If $E(tc)|_j$ is less than $E(tc)|_{j-1}$, set $j = j + 1$ and go to step 2; otherwise STOP ($n = j - 1$).

2.4 Conclusion

From the review of the literature, it can be concluded that all models in repeat multicharacteristic inspection were developed for a single inspection plan. The plan in the literature restricts the number of inspections for all characteristics to be equal. A need for a plan which allows variable number of inspections for different characteristics is felt.

This thesis proposes a new plan for multicharacteristic inspection and develops the optimization model which depicts the proposed plan. Algorithms and sequencing rules will be outlined and implemented. Incorporation of material handling cost explicitly in the inspection models will be carried out. This will be the subject discussed in the next chapters.

CHAPTER 3

DEVELOPMENT OF THE FIRST MODEL

3.1 Introduction

In this chapter a new development in repeat multicharacteristic inspection of critical components will be presented. This development is based on a new inspection plan. A model which depicts the plan is developed and provided in this chapter.

The plan in the literature is shown in Figure 2.1. In this plan each inspector inspects one characteristic and passes the accepted component to the next inspector who inspects the second characteristic. This process is repeated until all characteristics are inspected. It is assumed each inspector inspects one characteristic. All accepted components are sent back to the first inspector and the process is repeated until all the required number of inspections is completed.

In the new plan, it is proposed that the inspector inspects the characteristic the required number of times prior to passing it to the next inspector. Figure (3.1) illustrates the plan. In some situations this might be more suitable, especially in cases where defective rates of component characteristics vary and material handling cost is high.

Consideration of different plans is needed in order to select the most appropriate plan for the inspection. Each plan is evaluated based on the expected total cost of inspection. A complete model for each plan must be developed in order to optimize and obtain the expected total cost of inspection.

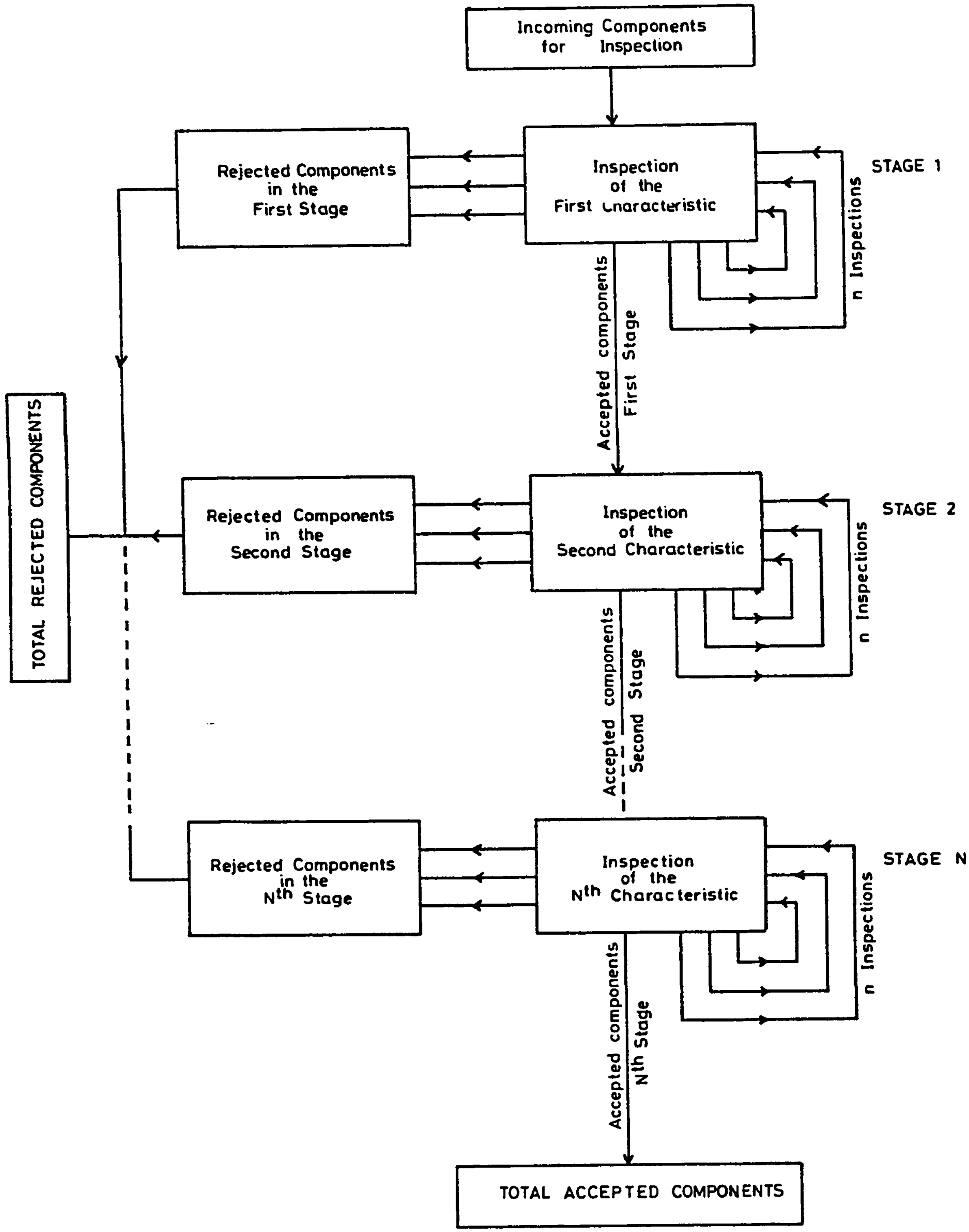

Fig. 3.1 Proposed Inspection Plan.

This chapter presents the model for the new plan and the algorithm proposed to obtain the optimal number of repeat inspections. Section 2 states the model assumptions. Section 3 presents the plan and the problem definition and section 4 provides the model development. Section 5 states computational procedures used to get the optimal number of repeat inspection. Section 6 presents illustrative examples. and section 7 concludes the chapter.

3.2 Model Assumptions

The model is developed for critical components having N characteristics for inspection with known incoming quality P_i. A component is classified as nondefective only if all the characteristics meet the quality specifications. The probabilities of type I error, P_{1i} and type II error, P_{2i} are assumed to be known. Three different types of costs are also considered:

(i) Cost due to false rejection of a non–defective component. C_r.

(ii) Cost due to false acceptance of a defective component. C_a.

(iii) Cost of inspection C_i.

The estimates for C_r, C_a, and C_i are assumed to be available in the industry.

3.3 Plan and Problem Definition

In order to control the quality of such critical components, the following inspection plan is proposed. The new inspection plan is applied as follows: The first inspector inspects one particular characteristic n times for each component entering the inspection process, (this is the first stage of inspection), and all the accepted components go to the second inspector, who inspects the second characteristic n times (this is the second stage of inspection). This chain of inspection continues until all the characteristics are inspected n times. Here n is the optimal number of inspections necessary to minimize the expected total inspection cost per accepted component. Each stage has n cycles of inspection. The new inspection plan was shown in Figure 3.1. Finally, the accepted components will be those which are accepted in the N-th stage, and the totality of rejected components is the sum of those rejected in the 1st, 2nd, ..., N-th stages. It is assumed that the arrival of components to the inspector is randomized when applying the inspection plans.

The objective is to find the optimal number of repeat inspections, n, to minimize the expected total cost. The expected total cost consists of the cost of false acceptance, cost of false rejection and cost of inspection. Next a mathematical model is developed to depict the proposed plan and to find the optimal number of repeat inspections.

3.4. Model Development

Prior to the model development, the following notation is adopted:

M_i	Number of components entering the i-th stage of inspection.
$M_{i,j}$	Number of components entering the j–th cycle of stage i.
N	Number of characteristics in each component to be inspected.
P_i	Probability of i-th characteristics in the sequence of inspection being defective entering the inspection.
PG	Probability of a component being non–defective entering the inspection.
P_{1i}	Probability of classifying the i-th nondefective characteristics in the sequence of inspection as defective (type I error).
P_{2i}	Probability of classifying the i-th defective characteristic in the sequence of inspection as nondefective (type II error).
${}^jP_i$	Probability of i-th characteristic in the sequence of inspection being defective entering the j-th cycle.
$PG(i,j)$	Probability of a component being nondefective entering the j-th cycle of the i-th stage of inspection.
$PG(i,n+1)$	Probability of a component being nondefective after inspecting characteristic i, n times.
$FR_{i,j}$	Number of falsely rejected components in the j–th cycle of the i–th stage.
$FA_{i,j}$	Number of falsely accepted components in the j–th cycle of the i–th stage.

$CA_{i,j}$	Number of correctly accepted components in the j–th cycle of the i–th stage.
${}^{j}R_{i}$	Rate of rejection of components due to i–th characteristic in the sequence of inspection of the j–th cycle.
$A(i)$	Number of accepted components in the i–th stage.
$CFR(i)$	Cost of false rejection in the i–th stage.
$CFA(i)$	Cost of false acceptance in the i–th stage.
$CI(i)$	Cost of inspection in the i–th stage.
$TCFR$	Total cost of false rejection.
$TCFA$	Total cost of false acceptance.
TCI	Total cost of inspection.
TA	Total number of accepted components.
$E(tc)\|_{j}$	Expected total cost per accepted component after j–th cycles of inspection.
$E(\quad)$	Expected value of the argument inside the parentheses.

Development of the Model

The probability of i-th characteristic being defective will vary from cycle to cycle. First, we shall establish the relationship between ${}^{j}P_{i}$ and P_{i}. The incoming quality, P_{i} is assumed to be known.

Expressing ${}^{j}P_i$ in Terms of Known P_i

Obviously

$$ {}^{1}P_i = P_i \tag{3.1} $$

Using Bayes' theorem,

$$ {}^{2}P_i = \frac{P_i P_{2i}}{[P_i P_{2i} + (1 - P_i)(1 - P_{1i})]} \tag{3.2} $$

Again

$$ {}^{3}P_i = \frac{{}^{2}P_i P_{2i}}{[{}^{2}P_i P_{2i} + (1 - {}^{2}P_i)(1 - P_{1i})]} $$

Substitution of ${}^{2}P_i$ from equation (3.2) into the above gives, after simplification,

$$ {}^{3}P_i = \frac{P_i P_{2i}^2}{[P_i P_{2i}^2 + (1 - P_i)(1 - P_{1i})^2]} \tag{3.3} $$

Similarly,

$$ {}^{4}P_i = \frac{P_i P_{2i}^3}{[P_i P_{2i}^3 + (1 - P_i)(1 - P_{1i})^3]} \tag{3.4} $$

In general, from the symmetry of the expression (3.2), (3.3) and (3.4),

$$ {}^{j}P_i = \frac{P_i P_{2i}^{j-1}}{\left[P_i P_{2i}^{j-1} + (1 - P_i)(1 - P_{1i})^{j-1}\right]} \tag{3.5} $$

The probability of a characteristic being defective changes in each cycle and so the probability for a component being nondefective will also change in each cycle. Bearing this in mind, we shall establish the relationship between $PG(i,j)$, the probability of a component being nondefective entering the j-th cycle of the i-th stage, and the incoming quality, P_i.

Expressing PG in Terms of P_i

The probability of a component being nondefective is

$$PG = \prod_{i=1}^{N}(1 - P_i) \tag{3.6}$$

$$PG(1,1) = PG = \prod_{i=1}^{N}(1 - P_i) \tag{3.7}$$

The probability of a component being nondefective after inspecting characteristics 1. n times

$$PG(1, n+1) = \left[\prod_{i=2}^{N}(1 - P_i)\right]\left[\left(1 - {}^{n+1}P_1\right)\right] \tag{3.8}$$

The probability of a component being nondefective after inspecting all characteristics n times

$$PG(N, n+1) = \prod_{i=1}^{N}\left(1 - {}^{n+1}P_i\right) \tag{3.9}$$

The probability of a component being nondefective after inspecting characteristic 1 through $i-1$, n times and characteristic i, k times and other characteristics from $i+1$ through N are not inspected is given by:

$$PG(i,k) = \left[\prod_{k=1}^{i-1}\left(1 - {}^{n+1}P_k\right)\right]\left[\left(1 - {}^{k+1}p_i\right)\right]\left[\prod_{k=i+1}^{N}(1 - P_k)\right] \tag{3.10}$$

When there is no inspection, the expected total cost per accepted unit will simply be the cost due to false acceptance of all defective components, *i.e.*,

$$E(tc)|_{j=0} = C_a(1 - PG) \tag{3.11}$$

where C_a is the cost of false acceptance per component and PG is given by equation (3.6).

The expected total cost per accepted component, after n cycles of inspections, can be written as

$$E(tc)_{j=n} = [TCFR + TCFA + TCI] / TA, \tag{3.12}$$

where $TCFR$, $TCFA$, TCI, and TA, are as defined earlier.

In order to determine $TCFR, TCFA, TCI$, and TA, an analysis of different stages of inspections is necessary.

Analysis of Stage (1)

All the components entering stage (1) go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. Figure 3.2 shows a complete stage of inspection. Each stage consists of n cycles. Following is the first cycle of inspection.

Cycle (1)

Number of components entering cycle 1 is

$$M_{1,1} = M_1 \tag{3.13}$$

The probability of a component being nondefective is

$$PG(1,1) = PG \tag{3.14}$$

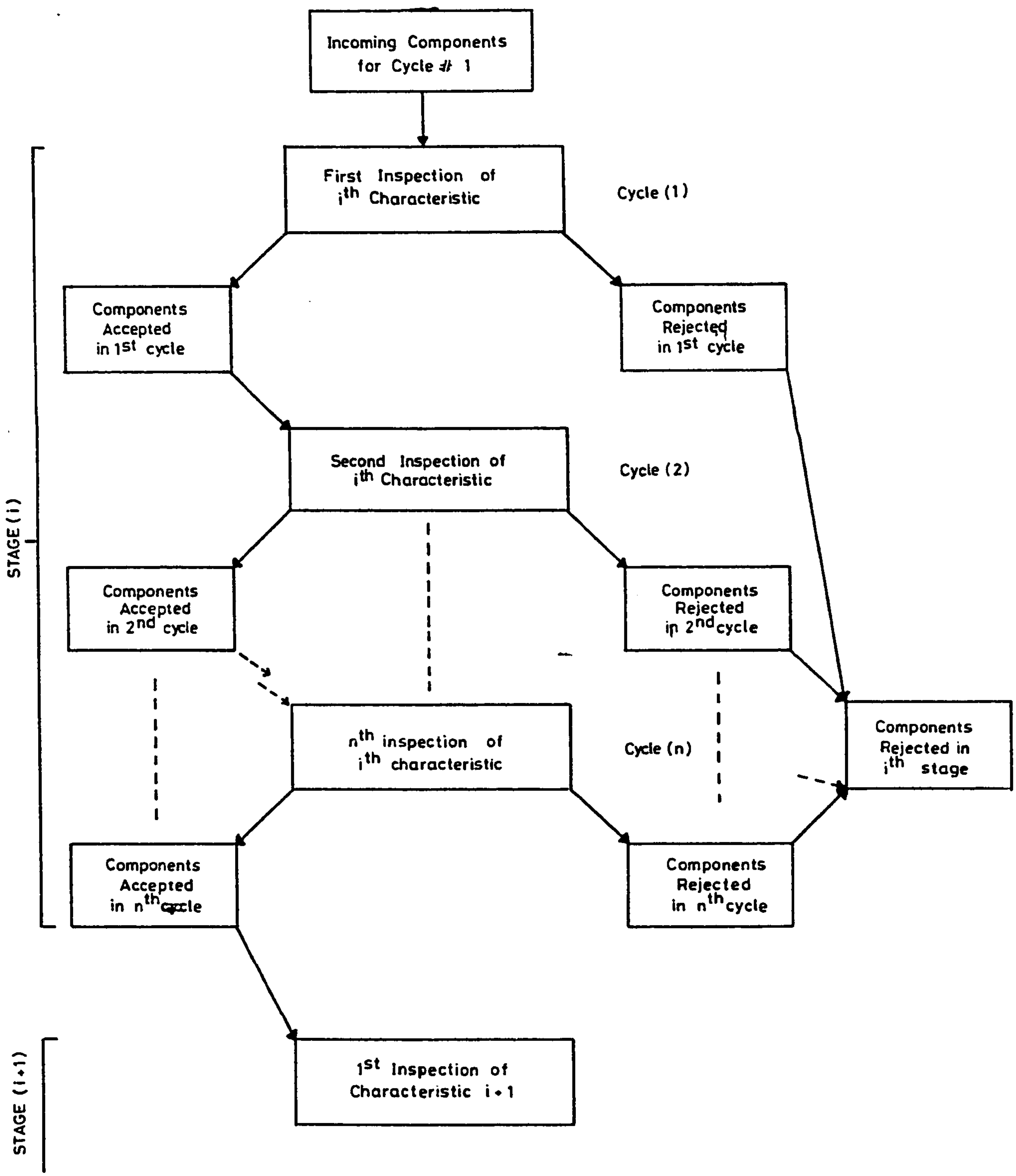

Fig. 3.2 Inspection plan for i^{th} stage. i = 1, 2, ..., N.

E (number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1} PG(1,1) P_{11} \\ &= M_1 PG\, P_{11} \end{aligned} \tag{3.15}$$

E (number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1} \left[P_1 P_{21} + (1 - PG(1,1) - P_1)(1 - P_{11}) \right] \\ &= M_1 \left[P_1 P_{21} + (1 - PG - P_1)(1 - P_{11}) \right] \end{aligned} \tag{3.16}$$

E (number of correctly accepted components) is

$$\begin{aligned} CA_{1,1} &= M_{1,1} PG(1,1)(1 - P_{11}) \\ &= M_1 PG(1 - P_{11}) \end{aligned} \tag{3.17}$$

All accepted components in this cycle go to the first inspector again to inspect the first characteristics for the second time for each component.

Cycle (2)

$$\begin{aligned} M_{1,2} &= FA_{1,1} + CA_{1,1} \\ &= M_1 \left[P_1 P_{21} + (1 - PG - P_1)(1 - P_{11}) \right] + M_1 PG(1 - P_{11}) \\ &= M_1 \left[P_1 P_{21} + (1 - P_1)(1 - P_{11}) \right] \end{aligned} \tag{3.18}$$

$$\begin{aligned} PG(1,2) &= CA_{1,1} / M_{1,2} \\ &= PG(1 - P_{11}) / \left[P_1 P_{21} + (1 - P_1)(1 - P_{11}) \right] \end{aligned} \tag{3.19}$$

$$FR_{1,2} = M_{1,2} PG(1,2) P_{11}$$

$$= M_1 PG P_{11}(1 - P_{11}) \tag{3.20}$$

$$FA_{1,2} = M_{1,2}\left[{}^2P_1P_{21} + (1 - PG(1,2) -{}^2 P_1)(1 - P_{11})\right]$$

$$= M_1\left[P_1P_{21} + (1 - P_1)(1 - P_{11})\right]\left[{}^2P_1P_{21} + \right.$$

$$\left. + (1 - PG(1,2) -{}^2 P_1)(1 - P_{11})\right] \tag{3.21}$$

$$CA_{1,2} = M_{1,2} PG(1,2)(1 - P_{11})$$

$$= M_1 PG(1 - P_{11})^2 \tag{3.22}$$

Similarly,

Cycle (3)

$$M_{1,3} = FA_{1,2} + CA_{1,2}$$

$$= M_1\left[P_1P_{21} + (1 - P_1)(1 - P_{11})\right]\left[{}^2P_1P_{21} + (1 - {}^2P_1)(1 - P_{11})\right] \tag{3.23}$$

$$PG(1,3) = CA_{1,2}/M_{1,3}$$

$$= PG(1 - P_{11})^2 / \left[P_1P_{21} + (1 - P_1)(1 - P_{11})\right] \times$$

$$\times\left[{}^2P_1P_{21} + (1 -{}^2 P_1)(1 - P_{11})\right] \tag{3.24}$$

$$FR_{1,3} = M_1 PG P_{11}(1 - P_{11})^2 \tag{3.25}$$

$$FA_{1,3} = M_1 \prod_{j=1}^{2}\left[{}^jP_1P_{21} + (1 -{}^j P_1)(1 - P_{11})\right] \times$$

$$\times\left[{}^3P_1P_{21} + (1 - PG(1,3) -{}^3 P_1)(1 - P_{11})\right] \tag{3.26}$$

$$CA_{1,3} = M_1 PG(1 - P_{11})^3 \tag{3.27}$$

Cycle (n)

From the symmetry of the expressions, we can write with the n-th cycle results:

$$M_{1,n} = M_1 \prod_{j=1}^{n-1} \left[{}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right] \tag{3.28}$$

$$PG(1,n) = PG(1 - P_{11})^{n-1} / \prod_{j=1}^{n-1} \left[{}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right] \tag{3.29}$$

$$FR_{1,n} = M_1 PG P_{11} (1 - P_{11})^{n-1} \tag{3.30}$$

$$\begin{aligned} FA_{1,n} = M_1 & \left[\prod_{j=1}^{n-1} \left\{ {}^{j}P_1 P_{21} + \left(1 - {}^{j}P_1\right)(1 - P_{11}) \right\} \right] \times \\ & \times \left[{}^{n}P_1 P_{21} + (1 - PG(1,n) - {}^{n}P_1)(1 - P_{11}) \right] \end{aligned} \tag{3.31}$$

$$CA_{1,n} = M_1 PG (1 - P_{11})^{n} \tag{3.32}$$

This completes one stage of inspection.

Results of Stage (1)

E (number of accepted components) is

$$\begin{aligned} A(1) &= FA_{1,n} + CA_{1,n} \\ &= M_1 \left[\prod_{j=1}^{n-1} \left\{ {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right\} \times \left\{ {}^{n}P_1 P_{21} + (1 - PG(1,n) - {}^{n}P_1) \times \right. \right. \\ & \quad \left. \left. \times (1 - P_{11}) \right\} + \left\{ PG(1 - P_{11})^{n} \right\} \right] \end{aligned} \tag{3.33}$$

where $PG(1,n)$ is given in equation (3.29).

Cost of false rejection after stage 1 is completed.

$$CFR(1) = C_r \sum_{j=1}^{n} FR_{1,j}$$

$$\begin{aligned} CFR(1) &= C_r M_1 PG \sum_{k=1}^{n} P_{11}(1-P_{11})^{k-1} \\ &= C_r M_1 PG P_{11} \sum_{k=1}^{n} (1-P_{11})^{k-1} \end{aligned} \tag{3.34}$$

Cost of false acceptance after stage 1 is completed.

$$\begin{aligned} CFA(1) &= C_a(FA_{1,n}) \\ &= C_a M_1 \left[\prod_{j=1}^{n-1} \left\{ {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1-P_{11}) \right\} \right] \times \\ &\quad \times [{}^{n}P_1 P_{21} + (1 - PG(1,n) - {}^{n}P_1)(1-P_{11})] \end{aligned} \tag{3.35}$$

Cost of inspection after stage 1 is completed

$$\begin{aligned} CI(1) &= C_1 \sum_{j=1}^{n} M_{1,j} \\ &= M_1 C_1 \left[\sum_{k=1}^{n} \prod_{j=1}^{k-1} \left\{ {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1-P_{11}) \right\} \right] \end{aligned} \tag{3.36}$$

E (total cost per accepted component after one stage of inspection) is:

$$E(tc)_{i=1} = [CFR(1) + CFA(1) + CI(1)] \,/\, [A(1)]$$

where $A(1), CFR(1), CFA(1)$, and $CI(1)$ are given by equations (3.33), (3.34), (3.35), and (3.36) respectively.

Analysis of Stage (2) of Inspection

Number of components entering stage 2 is $M_{2,1} = A(1)$ where $A(1)$ is given by equation (3.37).

Cycle (1)

$$M_{2,1} = FA_{1,n} + CA_{1,n}$$

$$= M_1 \prod_{j=1}^{n} \left[{}^jP_1P_{21} + (1 - {}^jP_1)(1 - P_{11})\right]$$

$$PG(2.1) = PG(1 - P_{11})^n / \left[\prod_{j=1}^{n} P_1P_{21} + (1 -^j P_1)(1 - P_{11})\right]$$

$$FR_{2,1} = M_1 PG\, P_{12}(1 - P_{11})^n$$

$$FA_{2,1} = M_1 \left[\prod_{j=1}^{n} \left\{{}^jP_1P_{21} + (1 -^j P_1)(1 - P_{11})\right\}\right] \times [P_2P_{22} + (1 - PG(2,1) - P_2) \times$$

$$\times (1 - P_{12})] + M_1PG(1 - P_{11})^n(1 - P_{12})$$

$$CA_{2,1} = M_1PG(1 - P_{11})^n(1 - P_{12})$$

Cycle (2)

$$M_{2,2} = FA_{2,1} + CA_{2,1}$$

$$= M_1 \left[\prod_{j=1}^{n} \left\{{}^jP_1P_{21} + (1 -^j P_1)(1 - P_{11})\right\}\right] \times$$

$$\times [P_2P_{22} + (1 - P_2)(1 - P_{12})]$$

$$PG(2,2) = PG(1 - P_{11})^n(1 - P_{12}) / \left[\prod_{j=1}^{n} \left\{{}^jP_1P_{21} + (1 -^j P_1)(1 - P_{11})\right\}\right] \times$$

$$\times [P_2P_{22} + (1 - P_2)(1 - P_{12})]$$

$$FR_{2,2} = M_1PGP_{12}(1 - P_{11})^n(1 - P_{12})$$

$$FA_{2,2} = M_1 \left[\prod_{j=1}^{n} \left\{ {}^jP_1P_{21} + (1 - {}^jP_1)(1 - P_{11} \right\} \right] \times$$
$$\times [P_2P_{22} + (1 - P_2)(1 - P_{12})]$$
$$\left[{}^2P_2P_{22} + (1 - PG(2,2) - {}^2P_2)(1 - P_{12}) \right]$$

$$CA_{2,2} = M_1PG(1 - P_{11})^n(1 - P_{12})^2$$

Cycle (n)

$$M_{2,n} = M_1 \left[\prod_{j=1}^{n} \left\{ {}^jP_1P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right\} \right] \times \left[\prod_{j=1}^{n-1} \left\{ {}^jP_2P_{22} + (1 - {}^jP_2)(1 - P_{12}) \right\} \right]$$

$$PG(2,n) = PG(1 - P_{11})^n(1 - P_{12})^{n-1} / \left[\prod_{j=1}^{n} \left\{ {}^jP_1P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right\} \right] \times$$
$$\times \left[\prod_{j=1}^{n-1} \left\{ {}^jP_2P_{22} + (1 - {}^jP_2)(1 - P_{12}) \right\} \right]$$

$$FR_{2,n} = M_1PGP_{12}(1 - P_{11})^n(1 - P_{12})^{n-1}$$

$$FA_{2,n} = M_1 \left[\prod_{j=1}^{n} {}^jP_1P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right] \times \left[\prod_{j=1}^{n-1} {}^jP_2P_{22} + (1 - {}^jP_2)(1 - P_{12}) \right] \times$$
$$\times [{}^nP_2P_{22} + (1 - PG(2,n) - {}^nP_2)(1 - P_{12})]$$

$$CA_{2,n} = M_1PG(1 - P_{11})^n(1 - P_{12})^n$$

Results of Stage (2)

$$A(2) = FA_{2,n} + CA_{2,n}$$
$$= M_1 \left[\prod_{j=1}^{n} {}^jP_1P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right] \times \left[\prod_{j=1}^{n-1} {}^jP_2P_{22} + (1 - {}^jP_2)(1 - P_{12}) \right] \times$$
$$\times [{}^nP_2P_{22} + (1 - PG(2,n) - {}^nP_2)(1 - P_{12})] + M_1PG(1 - P_{11})^n(1 - P_{12})^n$$

$$CFR(2) = C_r \sum_{j=1}^{n} FR_{2,j}$$
$$= C_r M_1 PGP_{12}(1 - P_{11})^n \sum_{k=1}^{n} (1 - P_{12})^{k-1}$$

$$CFA(2) = C_a(FA_{2,n})$$
$$= C_a M_1 \left[\prod_{j=1}^{n} {}^jP_1 P_{21}(1 -^j P_1)(1 - P_{11}) \right] \left[\prod_{j=1}^{n-1} {}^jP_2 P_{22} + (1 -^j P_2)(1 - P_{12}) \right]$$
$$[{}^nP_2 P_{22} + (1 - PG(2,n) -^n P_2)(1 - P_{12})]$$

$$CI(2) = C_2 \sum_{j=1}^{n} M_{2,j}$$
$$= M_1 C_2 \left[\prod_{j=1}^{n} {}^jP_1 P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right] \left[\sum_{k=1}^{n} \prod_{j=1}^{k-1} \left\{ {}^jP_2 P_{22} + (1 - {}^jP_2)(1 - P_{12}) \right\} \right]$$

Results needed to compute expected total cost

Total number of accepted components after inspecting the N–th characteristic:

$$A(N) = M \left[\prod_{k=1}^{N-1} \prod_{j=1}^{n} {}^jP_k P_{2k} + (1 - {}^jP_k)(1 - P_{1k}) \right] \times$$
$$\times \left[\prod_{j=1}^{n-1} {}^jP_N P_{2N} + (1 -^j P_N)(1 - P_{1N}) \right] \times$$
$$\times [{}^nP_N P_{2N} + (1 - PG(N,n) - {}^nP_N)(1 - P_{1N})] +$$
$$+ M \left[PG \prod_{k=1}^{N} (1 - P_{1k})^n \right] \tag{3.37}$$

Cost of false rejection in the i–th stage, $\quad i = 1, 2, \ldots, N$

$$CFR(i) = [C_r MPGP_{1i}] \left[\prod_{k=1}^{i-1} (1 - P_{1k})^n \right]$$
$$\times \left[\sum_{k=1}^{n} (1 - P_{1i})^{k-1} \right] \tag{3.38}$$

Cost of false acceptance after the N–th stage:

$$CFA(N) = C_a M \left[\prod_{k=1}^{N-1} \prod_{j=1}^{n} {}^{j}P_k P_{2k} + (1 -{}^{j} P_k)(1 - P_{1k}) \right] \times$$
$$\times \left[\prod_{j=1}^{n-1} {}^{j}P_N P_{2N} + (1 -{}^{j} P_N)(1 - P_{1N}) \right] \times$$
$$\times \left[{}^{n}P_N P_{2N} + (1 - PG(N,n) - {}^{n}P_N)(1 - P_{1N}) \right] \tag{3.39}$$

Cost of inspection at the i–th stage, $i = 1, 2, \ldots, N$

$$CI(i) = C_i M \left[\prod_{k=1}^{i-1} \prod_{j=1}^{n} {}^{j}P_k P_{2k} + (1 -{}^{j} P_k)(1 - P_{1k}) \right] \times$$
$$\times \left[\sum_{k=1}^{n} \left\{ \prod_{j=1}^{k-1} {}^{j}P_i P_{2i} + (1 -{}^{j} P_i)(1 - P_{1i}) \right\} \right] \tag{3.40}$$

Now, in order to determine the general expression for the expected total cost per accepted component, we must determine total cost of false rejection $TCFR$, total cost of false acceptance $TCFA$, total cost of inspection TCI, and total number of components finally accepted TA.

$$TCFR = \sum_{i=1}^{N} CFR(i) \tag{3.41}$$

$$TCFA = CFA(N) = C_a FA_{N,n} \tag{3.42}$$

$$TCI = \sum_{i=1}^{N} CI(i) \tag{3.43}$$

$$TA = A(N) = FA_{N,n} + CA_{N,n} \tag{3.44}$$

$$E(tc)_{j=n} = \frac{TCFA + TCFR + TCI}{TA} \tag{3.45}$$

The objective is to find the value of n which provides the minimum of $E(tc)_{j=n}$.

The probability of a component being non-defective entering the n-th cycle of inspection of the N-th characteristic is given by:

$$PG(N,n) = \left[\prod_{i=1}^{N-1}\left(1 - {}^{n+1}P_i\right)\right]\left[(1 - {}^{n}P_N)\right] \tag{3.46}$$

Number of components entering the j-th inspection of stage i is given by

$$M_{i,j} = M\left[\prod_{k=1}^{i-1}\prod_{j=1}^{n}\left\{{}^{j}P_k P_{2k} + (1 - {}^{j}P_k)(1 - P_{1k})\right\}\right]$$
$$\left[\prod_{k=1}^{j-1}\left\{{}^{k}P_i P_{2i} + (1 - {}^{k}P_i)(1 - P_{1i})\right\}\right] \tag{3.47}$$

Determining the Optimal Sequence of Inspection

The cost of inspection is influenced by the sequence in which characteristics are ordered for inspection, i.e., the order of stages. The following rule provides the optimal sequence of inspection for the characteristics, when each characteristic is inspected j times.

$$r(i) = \frac{C_i f_1({}^{j}R_i)}{1 - f_2({}^{j}R_i)} \quad \begin{array}{l} i = 1,2,\ldots,N. \\ j = 1,2,\ldots n. \end{array} \tag{3.48}$$

where

$${}^{j}R_i = {}^{j}P_i(1 - P_{2i}) + (1 - {}^{j}P_i)P_{1i}$$
$$f_1({}^{j}R_i) = \sum_{j=1}^{n}\left[\prod_{k=1}^{j}\left(1 - {}^{k-1}R_i\right)\right]$$
$$f_2({}^{j}R_i) = \prod_{k=1}^{n}\left(1 - {}^{k}R_i\right)$$

The characteristic with the lowest ratio is inspected first, next higher, second, next higher, third, and so on, and the characteristic with the highest ratio is the N-th characteristic to be inspected. The optimality of this rule follows from the proof given by Duffuaa and Raouf (1990). Next, a computational procedure is presented for finding the optimal n.

3.5 Computational Procedure

Step (1): Set $j = 0$, compute PG and $E(tc)_{j=0}$ using equations (3.6), (3.11) respectively.

Step (2): Let $j = j + 1$, sequence the characteristics according to equation (3.48).

Step (3): Compute ${}^{j}P_i$, $PG(N,n), A(N), CFR(i), CFA(N), CI(i)$ for all $i = 1, 2, \ldots, N$ from equations (3.5), (3.46), (3.37), (3.38), (3.39), and (3.40), respectively.

Step (4): Compute $TCFR, TCFA, TCI, TA$ and $E(tc)_j$ from equations (3.41), (3.42), (3.43), (3.44), and (3.45) respectively.

Step (5): If $E(tc)_j < E(tc)_{j-1}$, Go to 2, otherwise STOP $(n = j - 1)$.

3.6 Illustrative Examples

In order to compare the plan and the models in the literature with the proposed plan and models, the following examples are given. A software is developed implementing the algorithm in section (3.5) and is given in Appendix A. It is used to obtain the optimal number of repeat inspections.

Example (1)

Given the following known data

$M = 100$	$N = 3$	
$P_1 = .1$	$P_2 = .2$	$P_3 = .3$
$P_{11} = .01$	$P_{12} = .01$	$P_{13} = .01$
$P_{21} = .015$	$P_{22} = .015$	$P_{23} = .015$
$C_1 = 100$	$C_2 = 100$	$C_3 = 100$
	$C_a = 100,000$	
	$C_r = 500$	

Solving this example using Raouf *et al.*, [21] model gives the following result:

Optimal number of repeat inspections = 2
Minimum expected total inspection cost = 831.75
Optimal sequence $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99982

Solving the same example using the proposed model gives the following results:

Optimal number of repeat inspections = 2
Minimum expected total inspection cost = 880.90
Optimal sequence $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99982

Example (2)

Given

$M = 100$	$N = 3$	
$P_1 = .1$	$P_2 = .15$	$P_3 = .2$
$P_{11} = .01$	$P_{12} = .03$	$P_{13} = .015$
$P_{21} = 0.1$	$P_{22} = .09$	$P_{23} = .15$
$C_1 = 70$	$C_2 = 20$	$C_3 = 10$
	$C_a = 100,000$	
	$C_r = 500$	

Solving this example using Raouf *et al.*, [21] model gives:

Optimal number of repeat inspections = 3
Minimum expected total inspection cost = 560.81
Optimal sequence of inspection $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99886

Solving this example using the proposed model gives:

Optimal number of repeat inspections =3
Minimum expected total inspection cost = 555.57
Optimal sequence of inspection $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99886

The above two examples show that the proposed model in some situations provides slightly less expected total cost and therefore appears to be of use in these situations.

3.7 Results and Conclusions

A model based on the new plan is developed. An algorithm for obtaining the optimal number of repeat inspections is presented and implemented in a software. Two examples were given to demonstrate the new model. In the first example, Raouf *et al* (1983) model performed better in terms of expected total cost. In the second example, the proposed model showed a minor improvement over Raouf *et al* model.

Extensive testing and comparisons of the new model with the one in the literature will be carried out in chapter 5. In the next chapter, a general model will be developed. The general model allows for variable number of inspections for different characteristics.

CHAPTER 4

DEVELOPMENT OF THE SECOND MODEL

4.1 Introduction

This chapter presents a modification of the inspection plan given in chapter 3. The modified inspection plan with the model which represents it are given in this chapter.

In the modified new inspection plan it is suggested that the inspector inspects the first characteristic n_1 times. then passes the accepted components to the second inspector who inspects the second characteristic n_2 times, until all characteristics are inspected. $n_1, n_2, \ldots n_N$ are not necessarily equal.

The model for the modified plan is developed by relaxing the assumption of the equality of number of repeat inspections needed for each characteristic. This might be suitable in situations where the incoming quality of certain characteristics is different due to variations of machines capabilities in the production process.

This chapter presents the model for the modified plan and the algorithm proposed for obtaining the optimal number of repeat inspections for the given characteristics. Section 2 states the model assumptions. Section 3 presents the plan and the problem definition; section 4 provides the model development; section 5 provides computational procedures for obtaining the optimal number of repeat inspections; section 6 presents illustrative examples; and section 7 concludes the chapter.

4.2 Model Assumptions

The model is developed for critical components having N characteristics for inspection with known incoming quality P_i. A component is classified as nondefective only if all the characteristics meet the quality specifications. The probability of type I error, P_{1i}, and type II error, P_{2i} are assumed to be known. The different types of costs are also considered:

(i) cost due to false rejection of a nondefective component, C_r.

(ii) cost due to false acceptance of a defective component, C_a.

(iii) Cost of inspection C_i.

The estimates for C_r, C_a, and C_i are assumed to be available in the industry.

4.3 Plan and Problem Definition

In order to control the quality of such critical components, the following inspection plan is proposed. The modified inspection plan is applied as follows: The first inspector inspects one particular characteristic n_1 times for each component entering the inspection process; this is the first stage of inspection. All the accepted components go to the second inspector, who inspects the second characteristic n_2 times; this is the second stage of inspection. This chain of inspections continues until all the characteristics are inspected n_i times. Here n_i, $i = 1, 2, \ldots, N$, is the optimal number of inspections for characteristic i necessary to minimize the expected total cost of inspection per accepted component. The modified inspection plan is given in Figure 4.1. Finally, the accepted components will be those which are accepted in the N-th stage, and the totality of rejected components is the sum of those rejected in the 1st, 2nd, ..., N-th stages.

The objective is to find the optimal number of repeat inspections $n_1, n_2, \ldots, n_N$ to minimize the expected total inspection cost. The expected total cost consists of cost of false acceptance, cost of false rejection and cost of inspection.

Notations used are the same as those given in Chapter (3) except the new notation n_i which represents the number of repeat inspection for characteristic (i).

4.4 Model Development

Using the same formula used in chapter 3 to represent the probability of i-th characteristic being defective:

$$^{j}P_i = \frac{P_i P_{2i}^{j-1}}{P_i P_{2i}^{j-1} + (1 - P_i)(1 - P_{1i})^{j-1}} \tag{4.1}$$

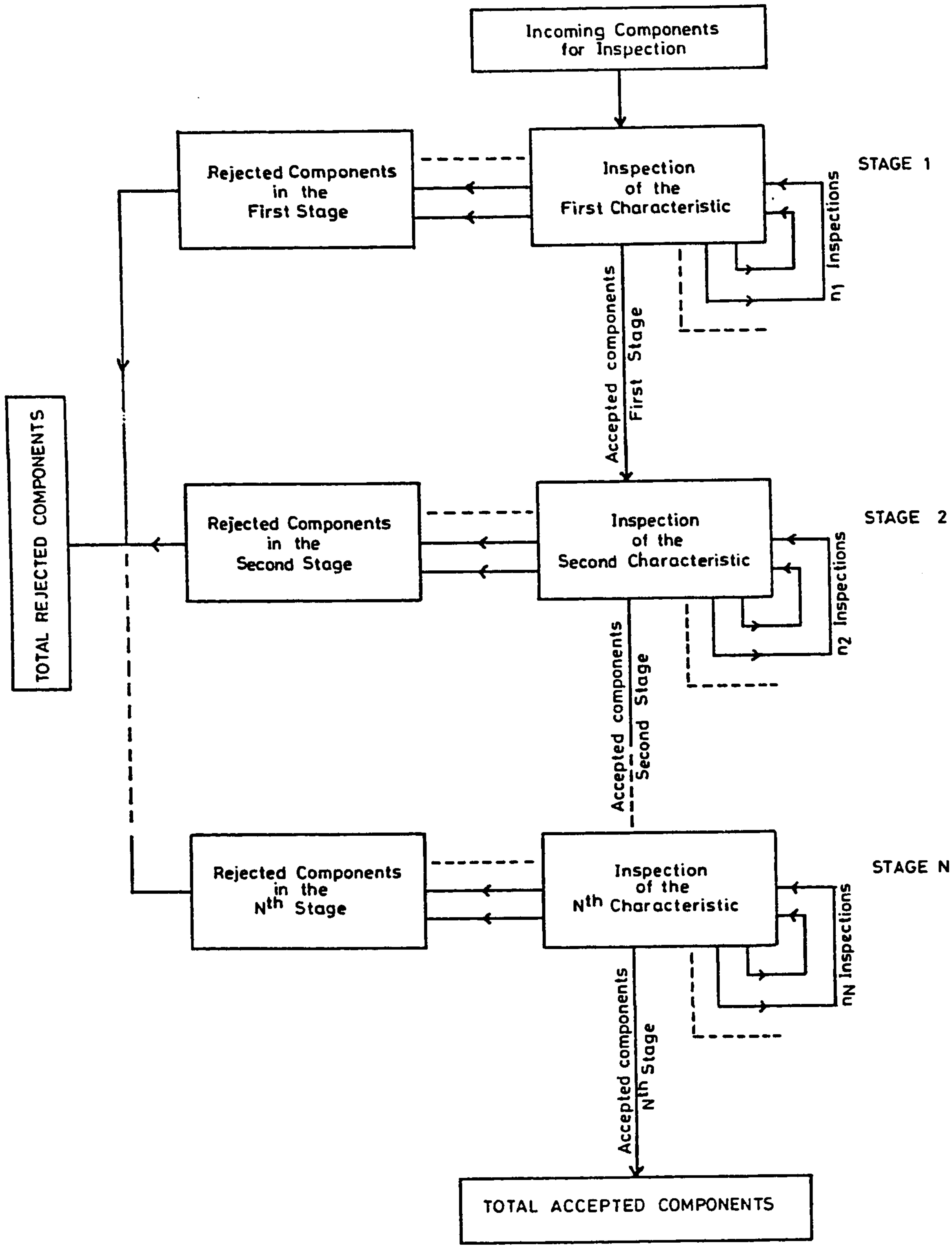

Fig.4.1 Modified proposed Inspection Plan.

Expressing PG in terms of P_i

The probability of a component being nondefective is

$$PG = \prod_{i=1}^{N}(1-P_i) \tag{4.2}$$

$$PG(1,1) = PG = \prod_{i=1}^{N}(1-P_i) \tag{4.3}$$

The probability of a component being nondefective after inspecting characteristic 1, n_1 times

$$PG(1,n_1+1) = \left[\prod_{i=2}^{N}(1-P_i)\right][(1-{}^{n_1}P_1)] \tag{4.4}$$

The probability of a component being nondefective after inspecting each characteristic n_i times

$$PG(N,n_i+1) = \prod_{i=1}^{N}\left(1-{}^{n_i+1}P_i\right) \tag{4.5}$$

The probability of a component being nondefective after inspecting characteristic 1 through $i-1$, n_i times and characteristic i, k times and other characteristics from $i+1$ through N are not inspected is given by

$$\begin{aligned} PG(i,k) = & \left[\prod_{k=1}^{i-1}\left(1-{}^{n_k+1}P_k\right)\right]\left[\left(1-{}^{k+1}P_i\right)\right] \\ & \left[\prod_{k=i+1}^{N}(1-P_k)\right] \end{aligned} \tag{4.6}$$

When there is no inspection, the expected total cost per accepted unit will simply be the cost due to false acceptance of all defective components, *i.e.*,

$$E(tc)]_{j=0} = C_a(1-PG) \tag{4.7}$$

where C_a is the cost of false acceptance per component and PG is given by equation (4.2).

The expected total cost per accepted component, after inspecting each characteristic n_i times can be written as

$$E(tc) = [TCFR + TCFA + TCI] / TA \tag{4.8}$$

where $TCFR, TCFA, TCI$, and TA, defined in chapter 3, need to be determined.

In order to determine $TCFR, TCFA, TCI$, and TA, an analysis of different stages of inspection is necessary.

Analysis of Stage (1)

All the components entering stage (1) go to the first inspector, who inspects the first characteristics in each component in order to classify it as defective or nondefective.

Cycle (n_1)

$$M_{1,n_1} = M_1 \left[\prod_{j=1}^{n_1-1} \left\{ {}^jP_1 P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right\} \right] \tag{4.9}$$

$$PG(1, n_1) = PG(1 - P_{11})^{n_1-1} \Big/ \left[\prod_{j=1}^{n_1-1} \left\{ {}^jP_1 P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right\} \right] \tag{4.10}$$

$$FR_{1,n_1} = M_1 PG P_{11} (1 - P_{11})^{n_1-1} \tag{4.11}$$

$$FA_{1,n_1} = M_1 \left[\prod_{j=1}^{n_1-1} \left\{ {}^jP_1 P_{21} + (1 - {}^jP_1)(1 - P_{11}) \right\} \right] \times$$

$$\times [{}^{n_1}P_1P_{21} + (1 - PG(1, n_1) - {}^{n_1}P_1)(1 - P_{11})] \tag{4.12}$$

$$CA_{1,n_1} = M_1PG(1 - P_{11})^{n_1} \tag{4.13}$$

This completes one stage of inspection.

Results of Stage (1)

E (number of accepted components) is

$$\begin{aligned} A(1) &= FA_{1,n_1} + CA_{1,n_1} \\ &= M_1 \left[\prod_{j=1}^{n_1-1} \left\{ {}^{j}P_1P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right\} \times \{ {}^{n}P_1P_{21} + \right. \\ &\quad \left. + (1 - PG(1, n_1) - {}^{n_1}P_1)(1 - P_{11})\} + \{PG(1 - P_{11})^{n_1}\} \right] \end{aligned} \tag{4.14}$$

where $PG(1, n_1)$ is given in equation (4.10).

Cost of false rejection after stage 1 is completed

$$\begin{aligned} CFR(1) &= C_r \sum_{j=1}^{n_1} FR_{1,j} \\ &= C_rM_1PG \sum_{k=1}^{n_1} \left[P_{11}(1 - P_{11})^{k-1} \right] \\ &= C_rM_1PGP_{11} \sum_{k=1}^{n_1} (1 - P_{11})^{k-1} \end{aligned} \tag{4.15}$$

Cost of false acceptance after stage 1 is completed

$$\begin{aligned} CFA(1) &= C_a(FA_{1,n_1}) \\ &= C_aM_1 \left[\prod_{j=1}^{n_1-1} \left\{ {}^{j}P_1P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right\} \right] \times \\ &\quad \times [{}^{n_1}P_1P_{21} + (1 - PG(1, n_1) - {}^{n_1}P_1)(1 - P_{11})] \end{aligned} \tag{4.16}$$

Cost of inspection after stage 1 is completed

$$\begin{aligned} CI(1) &= C_1 \sum_{j=1}^{n_1} M_{1,j} \\ &= M_1 C_1 \left[\sum_{k=1}^{n_1} \prod_{j=1}^{k-1} \left\{ {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right\} \right] \end{aligned} \quad (4.17)$$

E (total cost per accepted component after one stage of inspection) is:

$$E(tc)_{i=1} = [CFR(1) + CFA(1) + CI(1)]/[A(1)]$$

where $A(1), CFR(1), CFA(1)$, and $CI(1)$ are given by equations (4.14), (4.15), (4.16), and (4.17) respectively.

Analysis of Stage (2)

All the accepted components from stage (1) go to the second inspector, who inspects the second characteristic in each component in order to classify as defective or nondefective.

Cycle (n_2)

$$\begin{aligned} M_{2,n_2} = M_1 &\left[\prod_{j=1}^{n_1} \left\{ {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right\} \right] \times \\ &\times \left[\prod_{j=1}^{n_2-1} \left\{ {}^{j}P_2 P_{22} + (1 - {}^{j}P_2)(1 - P_{12}) \right\} \right] \end{aligned}$$

$$\begin{aligned} PG(2, n_2) = PG(1-P_{11})^{n_1}(1-P_{12})^{n_2-1} \Big/ &\left[\prod_{j=1}^{n_1} \left\{ {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11}) \right\} \right] \times \\ &\times \left[\prod_{j=1}^{n_2-1} \left\{ {}^{j}P_2 P_{22} + (1 - {}^{j}P_2)(1 - P_{12}) \right\} \right] \end{aligned}$$

$$FR_{2,n_2} = M_1 PG P_{12}(1-P_{11})^{n_1}(1-P_{12})^{n_2-1}$$

$$FA_{2,n_2} = M_1 \left[\prod_{j=1}^{n_1}\left\{{}^jP_1P_{21} + (1-{}^jP_1)(1-P_{11}\right\}\right]$$
$$\left[\prod_{j=1}^{n_2-1}\left\{{}^jP_2P_{22} + (1-{}^jP_2)(1-P_{12}\right\}\right]$$
$$[{}^{n_2}P_2P_{22} + (1-PG(2,n_2) - {}^{n_2}P_2)(1-P_{12})]$$

$$CA_{2,n_2} = M_1 PG(1-P_{11})^{n_1}(1-P_{12})^{n_2}$$

Results of Stage (2)

$$A(2) = FA_{2,n_2} + CA_{2,n_2}$$
$$= M_1 \left[\prod_{j=1}^{n_1}\left\{{}^jP_1P_{21} \div (1-{}^jP_1)(1-P_{11}\right\}\right]$$
$$\left[\prod_{j=1}^{n_2-1}\left\{{}^jP_2P_{22} + (1-{}^jP_2)(1-P_{12}\right\}\right]$$
$$[{}^{n_2}P_2P_{22} + (1-PG(2,n_2) - {}^{n_2}P_2)(1-P_{12})] +$$
$$+ M_1 PG(1-P_{11})^{n_1}(1-P_{12})^{n_2}$$

$$CFR(2) = C_r \sum_{j=1}^{n_2} FR_{2,j}$$
$$= C_r M_1 PG P_{12}(1-P_{11})^{n_1} \sum_{k=1}^{n_2}(1-P_{12})^{k-1}$$

$$CFA(2) = C_a(FA_{2,n})$$
$$= C_a M_1 \left[\prod_{j=1}^{n_1}\left\{{}^jP_1P_{21} + (1-{}^jP_1)(1-P_{11}\right\}\right]$$
$$\left[\prod_{j=1}^{n_2-1}\left\{{}^jP_2P_{22} + (1-{}^jP_2)(1-P_{12}\right\}\right]$$
$$[{}^{n_2}P_2P_{22} + (1-PG(2,n_2) - {}^{n_2}P_2)(1-P_{12})]$$

$$CI(2) = C_2 \sum_{j=1}^{n_2} M_{2,j}$$

$$= M_1 C_2 \left[\prod_{j=1}^{n_1} {}^{j}P_1 P_{21} + (1 - {}^{j}P_1)(1 - P_{11})\right]$$
$$\left[\sum_{k=1}^{n_2} \left\{\prod_{j=1}^{k-1} {}^{j}P_2 P_{22} + (1 - {}^{j}P_2)(1 - P_{12})\right\}\right]$$

Results needed to compute expected total cost

Total number of accepted components after inspecting the N–th characteristic

$$A(N) = M\left[\prod_{k=1}^{N-1}\prod_{j=1}^{n_k}({}^{j}P_k P_{2k}) + (1 - {}^{j}P_k)(1 - P_{1k})\right] \times$$
$$\times \left[\prod_{j=1}^{n_N-1} {}^{j}P_N P_{2N} + (1 - {}^{j}P_N)(1 - P_{1N})\right] \times$$
$$\times\, [{}^{n_N}P_N P_{2N} + (1 - PG(N, n_N) - {}^{n_N}P_N)(1 - P_{1N})] +$$
$$+ M\left[PG \prod_{k=1}^{N}(1 - P_{1k})^{n_k}\right] \tag{4.18}$$

Cost of false rejection in the i–th stage, $i = 1, 2, \ldots, N$

$$CFR(i) = C_r MPGP_{1i}\left[\prod_{k=1}^{i-1}(1 - P_{1k})^{n_k}\right]\left[\sum_{k=1}^{n_i}(1 - P_{1i})^{k-1}\right] \tag{4.19}$$

Cost of false acceptance after the N–th stage

$$CFA(N) = C_a M\left[\prod_{k=1}^{N-1}\prod_{j=1}^{n_k} {}^{j}P_k P_{2k} + (1 - {}^{j}P_k)(1 - P_{1k})\right] \times$$
$$\times \left[\prod_{j=1}^{n_N-1} {}^{j}P_N P_{2N} + (1 - {}^{j}P_N)(1 - P_{1N})\right] \times$$
$$\times\, [{}^{n_N}P_N P_{2N} + (1 - PG(N, n_N) - {}^{n_N}P_N)(1 - P_{1N})] \times \tag{4.20}$$

Cost of inspection at the i–th stage, $i = 1, 2, \ldots, N$

$$CI(i) = C_i M\left[\prod_{k=1}^{i-1}\prod_{j=1}^{n_k} {}^{j}P_k P_{2k} + (1 - {}^{j}P_k)(1 - P_{1k})\right] \times$$

$$\times \left[\sum_{k=1}^{n_i} \left\{ \prod_{j=1}^{k-1} {}^{j}P_i P_{2i} + (1 - {}^{j}P_i)(1 - P_{1i}) \right\} \right] \tag{4.21}$$

Now, in order to determine the general expression for the expected total cost per accepted component, we must determine total cost of false rejection $TCFR$, total cost of false acceptance $TCFA$, total cost of inspection TCI, and total number of components finally accepted TA.

$$TCFR = \sum_{i=1}^{N} CFR(i) \tag{4.22}$$

$$TCFA = CFA(N) = C_a FA_{N,n_N} \tag{4.23}$$

$$TCI = \sum_{i=1}^{N} CI(i) \tag{4.24}$$

$$TA = A(N) = FA_{N,n_N} + CA_{N,n_N} \tag{4.25}$$

$$E(tc) = \frac{TCFA + TCFR + TCI}{TA} \tag{4.26}$$

The objective is to find the value of n_i, $i = 1, 2, \ldots, N$ which provides the minimum of $E(tc)$.

The probability of a component being nondefective entering the n–th cycle of inspection of the N–th characteristic is given by

$$PG(N, n_N) = \left[\prod_{i=1}^{N-1} (1 - {}^{n_i+1}P_i) \right] [(1 - {}^{n_N}P_N)] \tag{4.27}$$

Number of components entering the j–th inspection of stage i is given by:

$$M_{i,j} = M \left[\prod_{k=1}^{i-1} \prod_{j=1}^{n_k} \left\{ {}^{j}P_k P_{2k} + (1 - {}^{j}P_k)(1 - P_{1k}) \right\} \right] \times$$

$$\times \left[\prod_{k=1}^{j-1} \left\{ {}^{k}P_i P_{2i} + (1 - {}^{k}P_i)(1 - P_{1i}) \right\} \right] \tag{4.28}$$

Determining the Optimal Sequence of Inspection

The optimal sequence of inspection for the characteristics, when each characteristic is inspected j times, can be obtained from the following ratio:

$$r(i) = \frac{C_i f_1({}^{j}R_i)}{1 - f_2({}^{j}R_i)} \qquad \begin{array}{l} i = 1, 2, \ldots, N \\ j = 1, 2, \ldots, n_i. \end{array} \tag{4.29}$$

where

$$\begin{aligned} {}^{j}R_i &= {}^{j}P_i(1 - P_{2i}) + (1 - {}^{j}P_i)P_{1i} \\ f_1({}^{j}R_i) &= \sum_{j=1}^{n_i} \left[\prod_{k=1}^{j} (1 - {}^{k-1}R_i) \right] \\ f_2({}^{j}R_i) &= \prod_{k=1}^{n_i} (1 - {}^{k}R_i) \end{aligned}$$

The characteristic with the lowest ratio is inspected first, next higher, second, next higher, third, and so on, and the characteristic with the highest ratio is the N-th characteristic to be inspected.

4.5 Computational Procedure

The computational procedure in this chapter depends on the concept of steepest descent.

At iteration i we have inspected characteristic i, r_i times. The total cost of inspection is

$$ETC(r_1, r_2, \ldots, r_N)$$

The descent in direction i, from $(r_1, r_2, \ldots, r_i, \ldots, r_N)$ to $(r_1, r_2, \ldots, r_i + 1, \ldots, r_N)$ is given by

$$DIN(i) = ETC(r_1, r_2, \ldots, r_i + 1, \ldots, r_N) - ETC(r_1, r_2, \ldots, r_N)$$

at each point the descent is computed in all directions. Then a move is made in the direction which has the largest descent. Suppose we are at the stage where each characteristic inspected r_i. The steps of the algorithm are:

STEP (1): Compute $ETC(r_1, r_2, \ldots, r_N)$.

STEP (2): FIND $DIN(i)$ for $i = 1, \ldots N$.

IF $DIN \geq 0$ for all i, GO TO STEP (6). Otherwise proceed.

STEP (3): FIND $\left\{ \max_i |DIN(i)| \;\middle|\; DIN(i) < 0 \right\} = DIN(k)$

STEP (4): Inspect characteristic k and compute $ETC(r_1, r_2, \ldots, r_k + 1, r_{k+1}, \ldots, r_N)$.

STEP (5): GO TO STEP (2).

STEP (6) STOP. The optimal inspection is $(r_1, r_2, \ldots, r_N)$ and the optimal total expected cost of inspection is $ETC(r_1, r_2, \ldots r_N)$.

The above stated algorithm is expected to provide a local minimum.

4.6 Illustrative Examples

In order to compare the model in the literature with the proposed model, the following examples are given. A software is developed implementing the algorithm in section (4.5) and is given in Appendix B. It is used to obtain the optimal number of repeat inspections.

Example (1)

Given the following known data:

$M = 100$	$N = 3$	
$P_1 = 0.01$	$P_2 = 0.2$	$P_3 = 0.3$
$P_{11} = .01$	$P_{12} = .01$	$P_{13} = .01$
$P_{21} = .015$	$P_{22} = .015$	$P_{23} = .015$
$C_1 = 100$	$C_2 = 100$	$C_3 = 100$
	$C_a = 100,000$	
	$C_r = 500$	

Solving this example using Raouf *et al.* [21] model gives the following results:

Optimal number of repeat inspections = 2
Minimum expected total inspection cost = 785.84
Optimal sequence $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99984

Solving the same example using the proposed model in Chapter (3) gives the following results:

Optimal number of repeat inspections = 2
Minimum expected total inspection cost = 812.052
Optimal sequence $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99984

Solving the same example using the modified model given in this chapter gives the following results:

Optimal number of repeat inspections
for characteristic 1 = 1
for characteristic 2 = 2
for characteristic 3 = 2
Minimum expected total inspection cost = 714.14
Optimal sequence $3 \rightarrow 2 \rightarrow 1$
Probability of a component being nondefective = 0.99969

Example (2)

Given

$M = 100$	$N = 3$	
$P_1 = .01$	$P_2 = .01$	$P_3 = 0.2$
$P_{11} = .01$	$P_{12} = .01$	$P_{13} = .01$
$P_{21} = .015$	$P_{22} = .015$	$P_{23} = .015$
$C_1 = 100$	$C_2 = 100$	$C_3 = 100$
	$C_a = 100,000$	
	$C_r = 500$	

Solving this example using Raouf *et al*, [21] model gives the following results:

Optimal number of repeat inspections = 2
Minimum expected total inspection cost = 692.54
Optimal sequence 3 → 1 → 2
Probability of a component being nondefective = 0.99994

Solving the same example using the proposed model in chapter (3) gives the following results:

Optimal number of repeat inspections = 2
Minimum expected total inspection cost = 694.832
Optimal sequence 3 → 1 → 2
Probability of a component being nondefective = 0.99994

Solving the same example using the modified model given in this chapter gives the following results:

Minimum number of repeat inspections
for characteristic 1 = 1
for characteristic 2 = 1
for characteristic 3 = 2
Minimum expected total inspection cost = 501.26
Optimal sequence 3 → 1 → 2
Probability of a component being nondefective = 0.99964

The above two examples show that the modified model provides less total expected cost.

4.7 Results and Conclusions

A general model based on the new plan is developed. An algorithm for obtaining the optimal number of repeat inspections is proposed and implemented in a software. Two examples were given to test the general model. The general model performed better in terms of expected total cost compared to the one in the literature. The saving in costs was 9.12% in the first example and 27.62% in the second example. The general model provides flexibillity of variable number of inspections for different characteristics. The variation of inspection is useful when having a characteristic with low value of P_i which implies less inspection for this characteristic compared with those characteristics with high value of P_i. The results of the two examples are promising. Extensive testing and comparison of the general model with the one in the literature will be covered in the next chapter.

CHAPTER 5

MODELS COMPARISON

5.1 Introduction

This chapter presents the results of comparing the models developed in this thesis with those in the literature. The comparison was carried out using randomly generated inspection problems. The results of the comparison indicate that model (2) in the thesis performed better in terms of expected total cost than Raouf *et al* model (1983). Model (1) in the thesis performed similar to the model in the literature. Details of the comparison are given in section 5.3. The rest of the chapter is organized as follows: Section 2 explains how the problems are generated; section 3 presents the models comparison; and section 4 concludes the chapter.

5.2 Random Problems Generation

The parameters of the problems generated are $N, C_i, C_a, C_r, P_i, P_{1i}$, and P_{2i}, for $i = 1, 2, \ldots, N$. The parameters N, C_i, C_r and C_a are all assumed to be uniformly distributed. While the parameters P_i, P_{1i} and P_{2i} are assumed to have normal distribution with know mean (u) and variance (σ^2).

The techniques used to generate random variables are obtained from [16] and the algorithms used in the generation are described in the next section. The random number used in the generation is of the linear congruential type.

Using the above concepts and assumptions, a computer software was developed, given in Appendix (C), to generate 100 problems for the inspection models in the thesis and the one in the literature.

5.2.1 Generating Number of Characteristics (N)

In each inspection model it was assumed that each component has N characteristics. The value of N is randomly generated from a discrete uniform distribution; and takes integer values $1, 2, \ldots, 10$. It is generated as follows:

a. Generate $U \sim U(0,1)$

b. Set $N = \lfloor a + (b * u) \rfloor$

where

$$a = 1 \quad \text{and} \quad b = 10,$$

$$U \quad \text{is a random number between } (0, 1)$$

and $\lfloor x \rfloor$ is the largest integer less than x.

5.2.2 Generating Different Inspection Costs

The expected total cost of inspection is a function of three types of costs. These costs are:

(i) cost of inspection, C_i.

(ii) cost of false acceptance, C_a.

(iii) cost of false rejection, C_r.

These three costs are randomly generated from a uniform distribution between a and b as follows:

Cost of inspection (C_i)

It is assumed that cost of inspection is uniformly distributed between a and b where $a = 10, \quad b = 100$.

a. Generate $U \sim (0,1)$

b. Set $C_i = a + (b - a)U \quad i = 1, 2, \ldots, N$.

Cost of False Acceptance (C_a)

a. Generate $U \sim (0,1)$

b. Set $C_a = a + (b - a)U$

where $a = 100,000$ and $b = 1,000,000$.

Cost of False Rejection (C_r)

a. Generate $U \sim (0,1)$

b. Set $C_r = a + (b-a)U$

where $a = 500$ and $b = 1,000$.

5.2.3 Generating Different Probabilities

In the inspection models there were three kinds of probabilities involved in the inspection process. These are:

(i) Probability of the i–th characteristic in the sequence of inspection being defective entering the inspection, P_i.

(ii) Probability of classifying the i–th nondefective characteristic in the sequence of inspection as defective (type I error), P_{1i}.

(iii) Probability of classifying the i–th defective characteristic in the sequence of inspection as nondefective (type II error), P_{2i}.

Generating P_i

It is assumed that the probability, P_i is normally distributed with mean, $u = 0.15$ and variance, $\sigma^2 = (0.048)^2$, and generated as follows:

1. Generate U_1 and U_2 as IID $U(0,1)$, let

$$V_1 = 2U_1 - 1$$

$$V_2 = 2U_2 - 1$$

$$W = V_1^2 - V_2^2$$

2. If $W > 1$, go back to step (1). Otherwise, let

$$Y = [(-2 \ln W)/W]^{1/2}$$

$$X_1 = V_1 Y$$

$$X_2 = V_2 Y$$

3. $P_i = 0.15 + 0.048X_1$, (for odd problems $(1, 3, \ldots, 99)$)

   $P_i = 0.15 + 0.048X_2$, (for even problems $(2, 4, \ldots, 100)$)

   for all $i = 1, 2, \ldots, N$

Generating P_{1i} (Probability of Type I Error)

It is assumed that the probability, P_{1i} is normally distributed with mean, $u = 0.1$ and variance, $\sigma^2 = (0.03)^2$, and generated as follows:

1. Generate U_1 and U_2 as IID $U(0,1)$, let

$$V_1 = 2U_1 - 1$$

$$V_2 = 2U_2 - 1$$

$$W = V_1^2 - V_2^2$$

2. If $W > 1$, go back to step (1). Otherwise, let

$$Y = [(-2\ln W)/W]^{1/2}$$

$$X_1 = V_1 Y$$

$$X_2 = V_2 Y$$

3. $P_{1i} = 0.1 + 0.03X_1$, (for odd problems (1, 3, ... 99))

   $P_{1i} = 0.1 + 0.03X_2$, (for even problems (2, 4, ... 100))

   for all $i = 1, 2, \ldots, N$

Generating P_{2i} (Probability of Type II Error)

It is assumed that the probability, P_{2i} is normally distributed with mean, $u = 0.1$ and variance, $\sigma^2 = (0.03)^2$, and generated as follows:

1. Generate U_1 and U_2 as IID $U(0,1)$, let

$$V_1 = 2U_1 - 1$$

$$V_2 = 2U_2 - 1$$

$$W = V_1^2 - V_2^2$$

2. If $W > 1$, go back to step (1). Otherwise, let

$$Y = [(-2\ln W)/W]^{1/2}$$

$$X_1 = V_1 Y$$

$$X_2 = V_2 Y$$

3. $P_{2i} = 0.1 + 0.03X_1$, (for odd problems (1, 3, ..., 99))

   $P_{2i} = 0.1 + 0.03X_2$, (for even problems (2, 4, ..., 100))

   for all $i = 1, 2, \ldots, N$

All the values of a's, b's, means and variances are based on experience. Using the above techniques, one hundred problems are generated for comparison purposes.

5.3 Models Comparison

The results of solving the randomly generated problems are summarized and given in Appendix (D). These results, using the three models are discussed in detail in this section. The criterion for better performance is the expected total cost of inspection.

5.3.1 Model (1) in the Thesis Versus Raouf et al Model

From the experiment conducted using the 100 randomly generated problems, it was found that model (1) gave better results in 46 problems out of 100. While Raouf *et al* model gave better results in 44 problems out of 100. The remaining 10 problems gave the same results using both models.

The reduction in cost of inspection when model (1) performed better in comparison with Raouf *et al* model ranges from 0.1% - 14%. While the reduction ranges from 0.1% to 12% when Raouf *et al* model performed better compared to model (1). A general conclusion can be drawn as follows:

Model (1) and Raouf *et al* model are almost similar.

5.3.2 Model (2) in the Thesis Versus Raouf et al Model

From the results given in Appendix (D), it was found that model (2) gave better results in 78 problems out of 100. While Raouf *et al* model gave better results in 12 problems out of 100. The remaining 10 problems gave similar results using both models.

The reduction in cost of inspection when model (2) performed better in comparison with Raouf *et al* model ranges from 1% to 20%. While the reduction ranges from 0.1% to 3% when Raouf *et al* model performed better compared to model (2).

General conclusions can be drawn as follows:

1. Model (2) in the thesis provides flexibility of variable number of inspections for different characteristics, where Raouf *et al* model does not have this property.

2. Due to the given property in (1), model (2) provides less total expected cost

than Raouf *et al* model in situations where the values of P_i's largely vary from one characteristic to another.

3. Raouf *et al* model tends to perform better or similar when there is small variation in the quality of incoming characteristics (*i.e.*, when the values of P_i's are identical or close to each other).

5.3.3 Model (2) in the Thesis Versus Model (1)

Model (2) and model (1) in the thesis have the same inspection plan. But model (2) differs from model (1) in terms of the number of repeat inspections for each characteristic. As mentioned earlier in the thesis, model (1) assumes an equal number of inspections for all characteristics. This assumption was relaxed in model (2) which allows for variable number of inspections for different characteristics. Due to this difference, model (2) provides less expected total cost than model (1). Also, model (2) is more suitable than model (1) in situations where variations of products because a variety of machines in the production process exist.

Given the results of the randomly generated problems in Appendix (D), model (2) gave better results in terms of expected total cost in 73 problems out of 100. In the rest of the 27 problems, both models gave equal results. Out of the 27 problems, 10 of the generated problems have one characteristic, which is an obvious situation where the two models should agree. It can be concluded that model (2) always performs better or similar to model (1). However, a situation might arise where model (1) gives better results than model (2), if the proposed algorithm for model (2) stopped at a local minimum.

The reduction in expected total cost of inspection when model (2) performed better in comparison with model (1) ranges from 0.1% to 25%.

The following table summarizes the results of comparing the two models in the thesis with the one in the literature based on the results of the 100 randomly generated problems.

	Percentage of problems where a model performed better in terms of expected total cost	Range of percentage reduction in expected total cost
Model (1) in the thesis versus Raouf *et al* model	46% model (1) is better 44% Raouf *et al* model 10% Equal	0.1% – 14% 0.1% – 12 % —
Model (2) in the thesis versus Raouf *et al* model	78% model (2) is better 12% Raouf *et al* model 10% Equal	1% – 20% 0.1% – 3% —
Model (2) versus Model (1) in the thesis	73% model (2) is better 27% Equal	0.1% – 25% —

Table 5.1 Summary of the Results

5.4 Conclusion

From the summarized results in Table (5.1), one can conclude that model (2) in the thesis gave better results and more savings in terms of expected total cost of inspection. This was expected due to the generality of the model, which allows for variable number of inspections for different characteristics. The saving in costs is significant in model (2) compared with the other two models.

Several batches of one hundred randomly generated problems were solved using model (2) and Raouf *et al* model. The percentages in which model (2) performed better than Raouf *et al* model are not sensitive and ranged between 75–80 percent. The savings in total expected cost ranges from 1–20 percent.

Sensitivity analyses were conducted for various values of mean, u and variance, $(\sigma)^2$ for the incoming quality, P_i. Values for mean, u range between .03 and .25. The results of running one hundred randomly generated inspection problems for different values of u, are summarized in Table 5.2. It was observed that at $u = 0.15$ model (1) in the thesis and Raouf *et al* model performed similar in terms of expected total cost of inspection. As the value of u decreases, model (1) performance improves and it reaches 88 percent at $u = 0.03$. However, the performance of Raouf *et al* model improves as u increases. The percentage of problems where Raouf *et al* model performs better in terms of expected total cost of inspection reaches 70 percent at $u = 0.25$.

u	σ	Percentage of problems where a model performed better in terms of expected total cost of inspection		
		Model (1) in the thesis versus Raouf *et al* model	Model (2) in the thesis versus Raouf *et al* model	Model (2) versus model (1) in the thesis
0.03	0.008	81% model (1) is better 3% Raouf *et al* model 16% Equal	88% model (2) is better 2% Raouf *et al* model 10% Equal	61% model (2) is better 39% Equal
0.05	0.012	82% Model (1) is better 4% Raouf *et al* model 14% Equal	87% Model (2) is better 3% Raouf *et al* model 10% Equal	65% Model (2) is better 35% Equal
0.10	0.03	67% Model (1) is better 23% Raouf *et al* model 10% Equal	86% Model (2) is better 4% Raouf *et al* model 10% Equal	68% Model (2) is better 32% Equal
0.15	0.48	46% Model (1) is better 44% Raouf *et al* model 10% Equal	78% Model (2) is better 12% Raouf *et al* model 10% Equal	73% Model (2) is better 27% Equal
0.2	0.064	32% Model (1) is better 58% Raouf *et al* model 10% Equal	76% Model (2) is better 14% Raouf *et al* model 10% Equal	74% Model (2) is better 26% Equal
0.25	.06	20% Model (1) is better 70% Raouf *et al* model 10% Equal	63% Model (2) is better 27% Raouf *et al* model 10% Equal	75% Model (2) is better 25% Equal

Table 5.2 Summary of the Sensitivity Analysis Results

The percentage of problems in which model (2) in the thesis performed better, compared to Raouf *et al* model, in terms of expected total cost of inspection ranges from 63 to 88 percent. It is noticed that as u decreases, the performance of model (2) improves. For exmaple, at $u = 0.03$, the percentage is 88, while the percentage reduces to 63 when $u = 0.25$.

In general, model (2) in the thesis performs better than Raouf *et al* model in terms of expected total cost of inspection. This performance improves when the incoming quality of characterisitcs is low.

CHAPTER 6

INCORPORATION OF MATERIAL HANDLING COST

6.1 Introduction

This chapter incorporates the material handling cost into the inspection models. The material handling cost is explicitly incorporated in these models to make them more realistic. In the literature, not much work has been done on material handling in inspection. In general, material handling cost is difficult - if not impossible - to assess or to determine accurately.

Material handling cost has been estimated for some production processes. It was cited in [1] that, in a typical manufacturing situation, processing and material handling may account for 50 percent of production costs, although this will vary considerably with the specific industry. The material handling portion will be about one–half of the total, or 25 percent of the total production cost, on the "average." However, this might vary from less than 10 percent to over 90 percent, depending on the particular situation [1].

Wang (1989) stated that "The concept of a quality transfer factor makes it possible to treat inspection stations and production stations equally." Borrowing this concept, material handling cost in inspection will be estimated as a portion of the inspection cost. This will be the subject of this chapter.

Details of the incorporation of material handling cost in inspection models will be shown in section 6.2. Section 6.3 presents the comparisons of the two models given in the thesis with the one given in the literature. The comparisons are based on the expected total cost of inspection with material handling consideration. Section 6.4 concludes this chapter.

6.2 Incorporation of Material Handling Cost in Inspection Models

It was mentioned in previous chapters that the expected total inspection cost consists of cost of false acceptance, cost of false rejection and inspection cost. Incorporating material handling cost in the expected total inspection cost will not affect the cost of false acceptance or the cost of false rejection. It will have an effect on the cost of inspection.

6.2.1 Incorporating Material Handling Cost in Raouf et al Model

The model description was given in chapter 2. The inspection plan was given in Figure 2.1. Based on the inspection plan, the following assumptions and notation are made and are given as follows:

From the plan shown in Figure 2.1, each characteristic is inspected in a different station. Accepted components are passed to next station for inspecting another characteristic. Given N characteristics in each component results in N inspection stations.

Let:

$r_{i-1,i}$: Distance travelled between station $i-1$ and station i.

$r_{N,1}$: Distance travelled between station N and station 1.

$\alpha_{i-1,i}$: The ratio of material handling cost between station $i-1$ and station i to the inspection cost at station i, C_i.

C_{ij}: Cost of inspection at j-th cycle of stage i.

See Figure 6.1.A for a case of three characteristics for the plan in Figure (2.1).

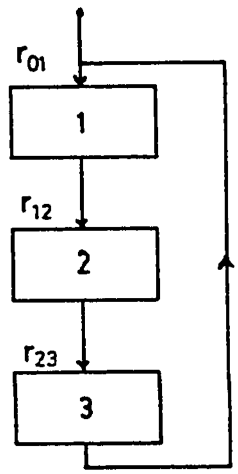

Fig. 6.1-A Material handling in the plan in Fig. 2.1.
(A case of three characteristics)

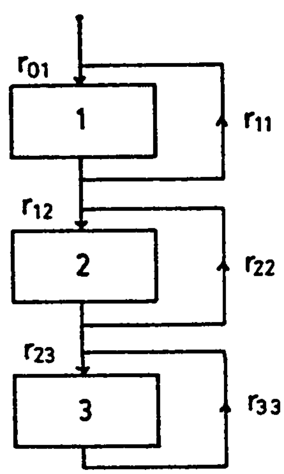

Fig. 6.1-B Material handling in the new plan in Fig. 3.1.
(A case of three characteristics)

The material handling cost is a function of distance travelled, type of handling equipment, type of product, inspection station layout, etc. In this case the cost of inspection consists of the cost of physical inspection C_i and the cost of material handling. For simplicity it is assumed that the cost of material handling can be estimated as a portion of the inspection cost as proposed by Wang (1989).

Cycle (1)

Each cycle consists of N inspection stations. The new inspection cost of characteristic i, will be the cost of physical inspection C_i, plus the cost of material handling between inspection stations $i-1$ and i.

Stage (1) [Station 1 of the first cycle]

$$C_{11} = C_1 + \alpha_{0,1} C_1$$

Stage (2) [Station 2 of first cycle]

$$C_{21} = C_2 + \alpha_{1,2} C_2$$

Stage (N) [Station N of the first cycle]

$$C_{N1} = C_N + \alpha_{N-1,N} C_N$$

Cycle (2)

Stage (1) [Station 1 of the 2nd cycle]

$$C_{12} = C_1 + \alpha_{N,1} C_1$$

Stage (2) [Station 2 of the 2nd cycle]

$$C_{22} = C_2 + \alpha_{1,2} C_2$$

Stage (N) [Station N of the 2nd cycle]

$$C_{N2} = C_N + \alpha_{N-1,N} C_N$$

Cycle (n)

Stage (1) [Station 1 of the n–th cycle]

$$C_{1n} = C_1 + \alpha_{N,1} C_1$$

Stage (2) [Station 2 of the n–th cycle]

$$C_{2n} = C_2 + \alpha_{1,2} C_2$$

Stage (N) [Station N of the n–th cycle]

$$C_{N,n} = C_N + \alpha_{N-1,N}$$

In summary,

$$C_{ij} = \begin{cases} C_1 + \alpha_{0,1} C_1 & \text{if } i = 1, n = 1 \\ C_1 + \alpha_{N,1} C_1 & \text{if } i = 1, n > 1 \\ C_i + \alpha_{i-i,i} C_i & \text{if } i > 1, \text{ for all } n \end{cases}$$

Equation (2.35) will be modified to be

$$CI(j) = \sum_{i=1}^{N} C_{i,j} M_{i,j} \tag{6.1}$$

Equation (6.1) calculates the cost of inspection in each cycle. Using equation (2.39), the expected total cost of inspection can be obtained. The formula for the expected total cost will not be changed, only the new value of TCI will be used in the formula.

For comparison purposes, the values of $\alpha_{i-1,i}$ are assumed to be a portion of the inspection cost as follows:

$$\alpha_{i-1,i} = 0.1 * C_i \quad \text{(10 percent of the inspection cost, } C_i\text{)}$$

$$\alpha_{N,1} = \frac{1}{2}\left[\alpha_{0,1} + \alpha_{1,2} + \cdots \alpha_{N-1,N}\right]$$

These values are functions of the distance travelled, type of material handling and equipment used in material handling.

6.2.2 Incorporating Material Handling Cost in the Two Models Given in the Thesis

The description of model (1) and model (2) in the thesis was given in chapters 3 and 4 respectively. The inspection plan was shown in Figure 3.1. Based on this plan the following assumptions and notations are made and are given as follows:

Let:

$r_{i-1,i}$: Distance travelled between station $i-1$ and station i.

$r_{i,i}$: Distance travelled within station i.

$\alpha_{i-1,i}$: Ratio of material handling cost between station $i-1$ and station i to the inspection cost of station i, C_i.

$\alpha_{i,i}$: Ratio of material handling within station i to cost of inspection C_i.

See Figure 6.2.B for a case of three characteristics

Given that, the new inspection cost will consist of two parts:

(i) physical inspection cost C_i.

(ii) material handling cost.

Applying these assumptions to the plan in the thesis:

Stage (1) [Station 1 of inspection]

Each stage consists of n inspections, see Figure 3.1. The new inspection cost is calculated as follows:

Cycle (1)

$$C_{11} = C_1 + \alpha_{0,1} C_1$$

Cycle (2)

$$C_{12} = C_1 + \alpha_{1,1} C_1$$

Cycle (n)

$$C_{1n} = C_1 + \alpha_{1,1}C_1$$

Stage (2) [Station 2 of inspection]

Cycle (1)

$$C_{21} = C_2 + \alpha_{1,2}C_2$$

Cycle (2)

$$C_{22} = C_2 + \alpha_{2,2}C_2$$

Stage (n)

$$C_{2n} = C_2 + \alpha_{2,2}C_2$$

Stage (N) [Station N of inspection]

Cycle (1)

$$C_{N,1} = C_N + \alpha_{N-1,N}C_N$$

Cycle (2)

$$C_{N,2} = C_N + \alpha_{N,N}C_N$$

Cycle (n)

$$C_{N,n} = C_N + \alpha_{N,N}C_N$$

In summary:

$$C_{ij} = \begin{cases} C_i + \alpha_{i-1,i}C_i & \text{for } j = 1,\ i = 1,2,\ldots,N \\ C_i + \alpha_{i,i}C_i & \text{for } j > 1,\ i = 1,2,\ldots N \end{cases}$$

Total inspection cost is calculated stage-wise and summed up over the stages. Total inspection cost at stage i is given by

$$CI(i) = \sum_{j=1}^{n_i} C_{ij} M_{i,j} \tag{6.2}$$

for model (1) $n_i = n$ for all $i = 1, 2, \ldots, N$ and the total inspection cost is given as

$$TCI = \sum_{i=1}^{N} CI(i) \tag{6.3}$$

The above change is implemented in the developed software and tested for the following values of $\alpha_{i,j}$.

For comparison purposes, the values of $\alpha_{i-1,i}$ and $\alpha_{i,i}$ are assumed to be a portion of the inspection cost as follows:

$\alpha_{i-1,i} = 0.1 * C_i$ (10 percent of the inspection cost, C_i)

$\alpha_{i,i} = 0.05 * C_i$ (5 percent of the inspection cost, C_i)

6.3 Models Comparison

The comparison was carried out using randomly generated inspection problems. Using the same scheme given in Chapter (5) and using the assumed values of α_{ij}, the results of solving the generated problems are summarized and given in Appendix (D). The criterion for better performance is the expected total inspection cost.

6.3.1 Model (1) in the Thesis versus Raouf et al Model

Using the results of the 100 randomly generated problems, it was found that model (1) gave better results in terms of expected total cost in 70 problems out of 100. While Raouf *et al* model gave better results in 20 problems out of 100. The remaining 10 problems gave equal results using both models.

The range of savings in the expected total cost of inspection when model (1) performed better in comparison with Raouf *et al* model ranges from 0.1% to 15%, while the savings range from 0.1% to 10% when Raouf *et al* model performed better compared to model (1).

6.3.2 Model (2) in the Thesis Versus Raouf et al Model

From the summary of the results in Appendix (D). it was found that model (2) performed better in terms of expected total cost in 86 problems out of 100, while Raouf *et al* model performed better in 4 problems. 10 problems gave equal results.

The savings in the total expected cost of inspection when model (2) performed better in comparison with Raouf *et al* model ranges from 1% to 27%. While the savings range from 0.1% to 3% when Raouf *et al* model performed better compared to model (2).

6.3.3 Model (2) in the Thesis Versus Model (1)

Given the results of randomly generated problems summarized in Appendix (D), model (2) performed better in terms of expected total cost of inspection in 73 problems out of 100. In the rest of the 27 problems, both models gave equal results. Out of the 27 problems, 10 of the generated problems have one characteristic, which is an obvious situation where the two models should agree. It can be concluded that model (2) always performs better or similar to model (1). However. a situation might arise where model (1) gives better results than model (2), if the proposed algorithm for model (2) stopped at a local minimum.

The reduction in expected total cost of inspection when model (2) performed better in comparison with model (1) ranges from 1% to 27%.

Table 6.1 summarizes the results of comparing the two models in the thesis with the one in the literature.

	Percentage of problems where a model performed better in terms of expected total cost	Range of percentage reduction in expected total cost
Model (1) in the thesis versus Raouf *et al* model	70% model (1) is better 20% Raouf *et al* model 10% Equal	0.1% – 15% 0.1% – 10 % —
Model (2) in the thesis versus Raouf *et al* model	86% model (2) is better 4% Raouf *et al* model 10% Equal	1% – 27% 0.1% – 3% —
Model (2) versus Model (1) in the thesis	73% model (2) is better 27% Equal	1% – 27% —

Table 6.1 Summary of the results with material handling

6.4 Conclusion

The summarized results given in Table 6.1 show that model (2) in the thesis performed better or equal results in most of the problems. (*i.e.* 96% compared to Raouf *et al* model (1983)). Savings of the expected total cost of inspection was significant in model (2) compared to the other two models.

The incorporation of material handling cost improved the performance of model (1) and model (2) in the thesis compared to Raouf *et al* model (1983). This was expected due to less material handling involvements in the new plan.

Increasing the portion of material handling cost in total inspection cost is in favor of plan (2). The summarized results based on the assumption that material handling cost ranges from 5% to 10% of the inspection cost. The above comparisons indicate that model (2) performed better and gave more savings in costs. Incorporating material handling cost made the three models more realistic. If model (1) performed better than model (2), it is due to the limitation in the solution methodology for the general model. The proposed method provides a local minimum.

From the summarized results in table (6.1), one can conclude that model (2) in the thesis performed better in terms of expected total cost of inspection. This was expected due to the generality of the model which allows variable number of inspections for different characteristics. The savings of cost is significant in model (2) compared to the other two models.

CHAPTER 7

CONCLUSIONS

7.1 Summary and Conclusions

The objective of this research was to extend the repeat multicharacteristic inspection plans in the literature. These plans require equal number of inspections for different characteristics. The work was motivated by the realization of a need to a new inspection plan which relaxes the assumption of equality of number of inspections and allows for variable number of inspections for different characteristics. The extensions of the research were based on a complete inspection plan given in the literature. The plan is instituted to guard against the human inspection errors. The implications of these errors could be catastrophic in the event of a critical component failure.

The new inspection plan for repeat multicharacteristic inspection of critical components was described and presented in chapter 3 of this thesis. A model to depict the proposed plan was developed and described. Computational procedures were suggested and a software implementing them was developed. In order to test the model and compare it to the one in the literature, randomly generated problems were used.

A more general model based on the new proposed plan which allows for variable number of inspections for different characteristics was presented in chapter 4. Also, computational procedures were suggested and a software implementing them was developed. To test the model and carry out comparisons, the same concept of randomly generated problems was utilized. Detailed comparisons were carried out. Conclusions of these comparisons were drawn in chapter 5.

Material handling cost was explicitly incorporated in the total inspection cost to make the models more realistic. The literature suggests that not much work has been done to incorporate the material handling cost in the total inspection cost. Hence, some assumptions were made to carry out the incorporation of material handling cost in inspection. The two models in the thesis were modified to incorporate material handling cost. Then comparisons between the models in the thesis and the ones in the literature were carried out.

The results of these comparisons were promising and encouraging. It was clear that model (2) in all aspects performed better in terms of cost and savings of cost compared to model (1) and Raouf *et al* model. In all situations, model (2) produced significant savings in expected total cost of inspection.

The results of this research can be applied to many practical situation where the cost of committing inspection error is substantial and therefore a need for repeat inspection is required. This research provides a complete inspection plan for critical components.

7.2 Future Research

The research here presented a new, complete inspection plan for critical components. Two models were developed to depict the plan. Some of the suggestions regarding possible research directions in the future are as follows:

1. Extending the two models given in this research to include the case where characteristics' defective rates are statistically dependent.

2. The effect of inspection errors can be studied in the case of the new plan.

3. Other objective functions can be considered. It can include, besides the minimization of expected total inspection cost, the maximization of the probability of a component being nondefective: in other words, considering multi–objective optimization.

4. The solution methodology proposed for the second model could be enhanced to provide global minimum.

5. The relationship between inspection cost and material handling need to be studied further.

6. Consideration of buffering strategies to smooth the process of inspection in the inspection plans needs to be investigated.

REFERENCES

1. Apple, J.M., Material Handling System Design, New York: John Wiley & Sons, 1972.

2. Ayoub, M.M.. Lambert, B.K. and Walkevar, A.G. "Effect of two types of inspection error on single sampling inspection", Presented at Human Factors Society Meeting. Baltimore, 1970.

3. Bennett, G.K.. Case, K.E.. Schmidt, J.W.. "The economic effects on inspector error on attribute sampling plans", Naval Research Logistics Quarterly, 21(3): 431–443, 1974.

4. Case, K.E.. Bennett, G.K. and Schmidt, J.W., "The effects of inspection errors on Average Outgoing Quality", Journal of Quality Technology, 7(1): 28–33, 1975.

5. Chandra J. and Schall, S., "The use of repeat measurements to reduce the effect of measurement errors", IIE Transactions. 20(1): 83–87, 1988.

6. Collins, R.D., Case, K.E. and Benett, G.K. "The effect of inspection accuracy in statistical quality control", Proc. 23rd Annual Conference and Convention, AIIE, Anaheim, CA, 1972.

7. Dorris, A.L., "The effects of inspection error on the operation of a C–chart", AIIE Transactions, 9(3): 312–315, 1977.

8. Duffuaa, S.O. and Nadeem, I., "Extensions in multicharacteristics repeat inspection plans", , unpublished master thesis, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia. 1993.

9. Duffuaa, S.O., and Raouf, A., "A cost minimization model for dependent multi–characteristic components", Proceedings of the 9th International Conference on Prod. Res. and Exhibition, Cincinnati, Ohio, USA, August 1987.

10. Duffuaa, S.O. and Raouf, A., "Mathematical optimization models for multicharacteristic repeat inspection", Applied Mathematical Modeling, (13): 408–411, 1989.

11. Duffuaa, S.O., and Raouf, A., "An optimal sequence in multicharacteristic inspection", Journal of Optimization Theory and Applications. (20): 79–86, 1990.

12. Duncan, A.J., Quality Control and Industrial Statistics, U.S.A.: Richard D. Irwin, Inc., 1974.

13. Elmagraby, S.E., "Comments on a DP Model for the Optimal Inspection Strategy", IEE Transactions, 18(1): 104–108, 1986.

14. Garcia–Diaz, A., Foster, J.W. and Bonguet, M., "Dynamic Programming Analysis of Special Multi–stage Inspectgion System", IEE Transaction 16(2): 115–126, 1983.

15. Harris, D.G., "Effect of defect rate on inspection accuracy", Journal of Applied Psychology, 52(5): 377–379, 1968.

16. Law, A.M. and Kelton, W.D., Simulation Modeling and Analysis. U.S.A: McGraw–Hill, Inc., 1982.

17. Lee, H.L., "On the optimality of a simplified multicharacteristic component inspection model", IEE Transaction, 20(4): 392–398, December 1988.

18. Maghsoodloo, S., "Inspection error effects on performance measures of multistage sampling plan", IEE Transactions, 19(3): 340–347, 1987.

19. Raouf, A. and Elfeturi, F., "A study of the effects of defect rate, task complexity and inspection rate on inspection accuracy", Proceedings of the First Saudi Engineering Conference, Jeddah, May 14–19, 1983.

20. Raouf, A., Jain, J.K., and P.T. Sathe, P.T., "A model for determining the optimal number of repeat inspections for complex units to minimize the total cost", Rep. Stat. Appl. Res., JUSE, 28(4): 121–131, December, 1981.

21. Raouf, A., Jain, J.K. and Sathe, P.T., "A cost minimization model for multicharacteristic component inspection", IIE Transactions, 187–194: Sep. 1983.

22. Tang, K. and Helmut S., "The effects of inspection error on a complete inspection plan", IIE Transaction, 19(4): 421–428, 1987.

23. Wallack, P.M., and Adams, S.K., "A comparison of inspection performance measures," AIEE Transactions, 2: 97–105, 1970.

24. Wang, L., "Inspection Effort Allocation in Serial Multistage Production Systems", Ph.D. Dissertation, The University of Iowa, 1989.

25. Yumei, L. and Tang, T., "Economic design of multicharacteristc models for three-class screening procedure', 'International Journal of Production Research, 28(12): 2341–2351, 1990.

APPENDICES

APPENDIX (A)

```
C
C
C
C
C
C                      ***************************
C                      ***                     ***
C                      ***      APPENDIX A     ***
C                      ***                     ***
C                      ***************************
C
C
C*************************************************************************
C********                                                        ********
C********      A LISTING OF THE PROGRAM USED TO COMPUTE           ********
C********      THE TOTAL EXPECTED COST OF INSPECTION              ********
C********      USING MODEL ( 1 ) IN THE THESIS.                   ********
C********                                                        ********
C*************************************************************************
C
C
C
C ***********************************************************************
      REAL P(40,0:40),PGS(0:40),RR(40,40)
      REAL P1(40), P2(40), CII(40), ETC(0:40)
      REAL OP1(40), OP2(40), OCII(40),OP(40,0:40)
      REAL R(40), A(40), CFR(40), CI(40), SEQ(40),SEK(40)
      REAL MH(40),H(40)
      OPEN(17,FILE='MODEL-1 DATA A')
C
C ***********************************************************************
C *****************       READING INPUT DATA      ***********************
C ***********************************************************************
C
      READ*, M,N
      READ*,(P(I,0),I=1,N)
      READ*,(P1(I),I=1,N)
      READ*,(P2(I),I=1,N)
      READ*,(CII(I),I=1,N)
      READ*,CFAI
      READ*,CFRI
      READ*,(H(I),I=1,N)
C
C ***********************************************************************
C ******************      INITILIZATION           ******************
C ***********************************************************************
C
        DO 798 I=1,N
        OP(I,0)=P(I,0)
        OP1(I) =P1(I)
        OP2(I) =P2(I)
        OCII(I)=CII(I)
 798    CONTINUE
C
C ***********************************************************************
```

```
C ******************     NO INSPECTION (JJ=0)      ******************
C *******************************************************************
C
      JJ = 0
      PG = 1
      DO 10 I = 1, N
         SEQ(I) = I
         PG = PG * (1 - P(I,0))
         P(I,1)=P(I,0)
 10   CONTINUE
      PGS(0)=PG
C
C *******************************************************************
C ******************     COST WHEN JJ=0        ******************
C *******************************************************************
C
      ETC(JJ) = CFAI* (1-PG)
      WRITE(17,*)'-------------------------------------------------'
      WRITE(17,*)'  PG ( 0 ) = ',PG
      WRITE(17,151)ETC(JJ)
 151  FORMAT(2X,'NO INSPECTION COST (JJ=0) = ',F14.3)
      WRITE(17,*)'-------------------------------------------------'
C
C *******************************************************************
C ******************     COST WHEN JJ > 0      ******************
C *******************************************************************
C
 20   JJ = JJ + 1
C--------------------     STARTING THE INSPECTION     -----------------
       DO 71 I=1,N
       SEK(I) = SEQ(I)
       SEQ(I) = I
       P(I,0)=OP(I,0)
       P(I,1)=P(I,0)
       P1(I) =OP1(I)
       P2(I) =OP2(I)
       CI(I)=OCI(I)
  71   CONTINUE
C-------------------------------------------------------------------
      WRITE(17,*) '    **********************                 '
      WRITE(17,111)JJ
 111  FORMAT(6X ,'# OF INSP. ===> ',I3)
      WRITE(17,*) '    **********************                 '
C
C *******************************************************************
C ******************  UPDATING THE P (I,JJ)    ******************
C *******************************************************************
C
       IF (JJ .GE. 1) THEN
       DO 22 I = 1, N
       DO 33 L = 1,JJ+1
         S1 = P(I,0) * P2(I)**(L-1)
         S2 = (1-P(I,0)) * (1-P1(I))**(L-1)
         P(I,L) = S1/(S1 + S2)
  33   CONTINUE
```

```
  22  CONTINUE
      ENDIF
C
C ****************************************************************************
C *******************      THE SEQUENCING RULE       *****************
C ****************************************************************************
C
      DO 48 I=1,N
      DO 48 J=1,JJ
48    RR(I,J)=(P(I,J)*(1-P2(I)))+((1-P(I,J))*P1(I))
     DO 30 I = 1, N
      F1 = 1
      DO 40 J = 2 ,JJ
        S = 1
        DO 50 K = 2, J
 50       S = S * (1-RR(I,K-1))
 40     F1 = F1 + S
      F2  = 1
      DO 60 J = 1, JJ
 60     F2 = F2 * ( 1 - RR(I,J))
 30   R(I) = (CII(I)* F1 )/(1-F2)
      DO 66 I = 1, N
 66   SEQ(I)=SEQ(I)
      DO 70 I = 1, N
      DO 70 K = 1, N-1
        IF (R(K) .GT. R(K+1) ) THEN
      CALL SWAP(R(K),R(K+1))
         DO 44 J=0,JJ+1
 44   CALL SWAP(P(K,J),P(K+1,J))
      CALL SWAP(P1(K),P1(K+1))
      CALL SWAP(P2(K),P2(K+1))
      CALL SWAP(CII(K),CII(K+1))
      CALL SWAP(SEQ(K),SEQ(K+1))
      ENDIF
 70   CONTINUE
      WRITE(17,*) '----------------------------------'
      WRITE(17,112)JJ
 112  FORMAT(1X,'THE SEQ. OF THE CHAR. ',I3)
      WRITE(17,*) '----------------------------------'
      DO 77 I=1,N
 77   WRITE(17,*) INT( SEQ(I))
      WRITE(17,*) '----------------------------------'
C
C ****************************************************************************
C *******************      HOW TO CALCULATE A(N)       *****************
C ****************************************************************************
C
        S2=1
        DO 238 I=1,N
        S3=(1-P1(I))**JJ
 238    S2=S2 * S3
        S6=S2 * PG
       PGN = 1
       DO 235 I = 1, N-1
 235     PGN = PGN * ( 1 - P(I,JJ+1))
```

```
         PGN = PGN * ( 1 - P(N,JJ))
         S1 =  P(N,JJ) * P2(N) + (1 - PGN - P(N,JJ))* (1-P1(N))
            S2 = 1
         DO 221 K = 1, N-1
            S3 = 1
           DO 222 J = 1, JJ
            S4 = P(K,J) * P2(K)
            S5 = (1-P(K,J)) * (1 - P1(K))
 222        S3 = S3 * ( S4 + S5)
 221     S2 = S2 * S3
            S3 = 1
            DO 233 J = 1, JJ-1
              S4 =  P(N,J) * P2(N)
              S5 = (1-P(N,J)) * (1-P1(N))
 233     S3 = S3 * (S4 + S5)
      A(N) =  M *( S1 * S2 * S3 + S6 )
C
C ********************************************************************
C ******************    HOW TO CALCULATE CFR         ******************
C ********************************************************************
C
       DO 100 I = 1, N
         S1 = 1
         DO 101 K = 1, I-1
 101       S1 = S1 * (1-P1(K))**JJ
         S2 = 0
         DO 102 K = 1, JJ
 102       S2 = S2 + ( 1 - P1(I))**(K-1)
 100     CFR(I) = S1 * S2 * P1(I) * PG * M * CFRI
C
C ********************************************************************
C ******************    HOW TO CALCULATE CFA         ******************
C ********************************************************************
C
         PGN = 1
         DO 200 I = 1, N-1
 200       PGN = PGN * ( 1 - P(I,JJ+1))
         PGN = PGN * ( 1 - P(N,JJ))
         S1 =  P(N,JJ) * P2(N) + (1 - PGN - P(N,JJ))* (1-P1(N))
            S2 = 1
         DO 201 K = 1, N-1
            S3 = 1
           DO 202 J = 1, JJ
            S4 = P(K,J) * P2(K)
            S5 = (1-P(K,J)) * (1 - P1(K))
 202        S3 = S3 * ( S4 + S5)
 201     S2 = S2 * S3
            S3 = 1
            DO 203 J = 1, JJ-1
              S4 =  P(N,J) * P2(N)
              S5 = (1-P(N,J)) * (1-P1(N))
 203     S3 = S3 * (S4 + S5)
      CFAF = S1 * S2 * S3   * M * CFAI
           IF(CFAF .LT. 0.0)THEN
           CFAF=0.0
```

```
          ENDIF
C
C ********************************************************************
C ******************    HOW TO CALCULATE CI(I)      ******************
C ********************************************************************
C
      DO 300 I = 1, N
          S1=0.0
          DO 310 K=1,JJ
          S2 = 1
          DO 320 J = 1, K-1
            S4 = P(I,J) * P2(I)
            S5 = (1-P(I,J)) * (1-P1(I))
 320        S2 = S2 * (S4 + S5)
 310        S1 = S1+S2
        S2=1
      DO 400 L = 1,I-1
        S3=1
        DO 410 J = 1, JJ
          S4 = P(L,J) * P2(L)
          S5 = (1-P(L,J)) * (1-P1(L))
 410      S3 = S3 * (S4 + S5)
 400    S2 = S2 * S3
 300    CI(I) =  CII(I) * M  *  S2  * S1
C
C ********************************************************************
C ******************    HOW TO CALCULATE MH(I)      ******************
C ********************************************************************
C
      DO 337 I = 1, N
          S1=1.0
          DO 311 K=1,JJ
          DO 323 J = 1, K-1
            S4 = P(I,J) * P2(I)
            S5 = (1-P(I,J)) * (1-P1(I))
 323        S1 =  S1 * (S4 + S5)
        S2=1
      DO 408 L = 1,I-1
        S3=1
        DO 413 J = 1, JJ
          S4 = P(L,J) * P2(L)
          S5 = (1-P(L,J)) * (1-P1(L))
 413      S3 = S3 * (S4 + S5)
 408    S2 = S2 * S3
        IF ( K .EQ. 1) THEN
        MH(I) =  H(I) * CII(I) * M  *  S2
        ELSE
        MH(I) = (.5 * H(I) * CII(I) * M * S1 * S2) + MH(I)
        ENDIF
 311    CONTINUE
 337    CONTINUE
C
C ********************************************************************
C ******************    PG AFTER INSPECTION         ******************
C ********************************************************************
```

23C

```
C
      PGS(JJ)=1
         DO 488 I=1,N
 488     PGS(JJ) =PGS (JJ) * ( 1 - P(I,JJ+1))
         WRITE(17,1)JJ,PGS(JJ)
  1      FORMAT(2X,'PG  (',I2,' ) = ',5X,F14.8)
C
C ******************************************************************
C ******************       HOW TO CALCULATE TCFR        ******************
C ******************************************************************
C
      TCFR = 0
      DO 500 I = 1, N
 500    TCFR = TCFR + CFR(I)
      WRITE(17,2)JJ,TCFR
  2      FORMAT(2X,'TCFR(',I2,' ) = ',F14.3)
C
C ******************************************************************
C ******************       HOW TO CALCULATE TCFA        ******************
C ******************************************************************
C
      TCFA = CFAF
      WRITE(17,3)JJ,TCFA
  3      FORMAT(2X,'TCFA(',I2,' ) = ',F14.3)
C
C ******************************************************************
C ******************       HOW TO CALCULATE TCI         ******************
C ******************************************************************
C
      TCI = 0
      DO 600 I = 1, N
 600     TCI = TCI + CI(I)
      WRITE(17,4)JJ,TCI
  4      FORMAT(2X,'TCI (',I2,' ) = ',F14.3)
C
C ******************************************************************
C ******************       HOW TO CALCULATE TCMH        ******************
C ******************************************************************
C
      TCMH = 0
      DO 615 I = 1, N
 615     TCMH = TCMH + MH(I)
      WRITE(17,8)JJ,TCMH
  8      FORMAT(2X,'TCMH(',I2,' ) = ',F14.3)
C
C ******************************************************************
C ******************       HOW TO CALCULATE TA          ******************
C ******************************************************************
C
      TA = A(N)
      WRITE(17,5)JJ,A(N)
  5      FORMAT(2X,'TA  (',I2,' ) = ',F14.3)
C
C ******************************************************************
C ******************       HOW TO CALCULATE ETC(JJ)     ******************
```

```
C ********************************************************************
C
      ETC (JJ) = ( TCFR + TCFA + TCI + TCMH)/TA
      WRITE(17,6)JJ,ETC(JJ)
 6       FORMAT(2X,'ETC (',I2,' ) = ',F14.3)
      WRITE(17,*)'  '
      WRITE(17,*) '---------------------------------'
      WRITE(17,*)'  '
C
C ********************************************************************
C ******************    STOPPING CRITERION         *****************
C ********************************************************************
C
      IF (ETC(1)  .GE. ETC(0)   ) THEN
      WRITE(17,*)'OPTIMAL SOLUTION :  '
      WRITE(17,*)'------------------  '
      WRITE(17,*)'****  IT IS BETTER NOT TO DO INSPECTION   **** '
      WRITE(17,*)'                                '
       GO TO 99
      ENDIF
      IF (ETC(JJ) .LT. ETC(JJ-1)) GOTO 20
C
C ********************************************************************
C ******************    PRINTING THE RESULTS       *****************
C ********************************************************************
C
      WRITE(17,*)'OPTIMAL SOLUTION :'
      WRITE(17,*)'------------------'
      WRITE(17,*)'                          '
      WRITE(17,*)'PG              =',PGS(JJ-1)
      WRITE(17,*)'MIN # OF INSP. =    ',JJ-1
      WRITE(17,*)'MIN COST        =',ETC(JJ-1)
      WRITE(17,*)'OPTIMAL SEQ.     '
      WRITE(17,*)'---------------'
      WRITE(17,990)(INT(SEK(I)),I=1,N)
 990  FORMAT   (5X, I3)
 99   WRITE(17,*)'======================================================'
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'-----------   NO ENTRIES BEYOND THIS LINE  ---------'
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      WRITE(17,*)'                                                      '
      CLOSE(17)
      END
C
C********************************************************************
C*******  THIS SUBROUTINE IS TO SWAP THE CHARACTERISTICS        ******
C*******  WITH THERE ASSOCIATED DATA.                           ******
C********************************************************************
```

```
C
      SUBROUTINE SWAP(A,B)
      T = A
      A = B
      B = T
      END
C$DATA
100   3
0.1     0.2     0.3
0.01    0.01    0.01
0.015   0.015   0.015
100     100     100
100000
500
0  0  0
```

APPENDIX (B)

```
C
C
C
C
C
C                    **************************
C                    ***                    ***
C                    ***    APPENDIX B      ***
C                    ***                    ***
C                    **************************
C
C*********************************************************************
C********                                                     ********
C********     A LISTING OF THE PROGRAM USED TO COMPUTE        ********
C********     THE TOTAL EXPECTED COST OF INSPECTION           ********
C********     USING MODEL ( 2 ) IN THE THESIS.                ********
C********                                                     ********
C*********************************************************************
C
C
      INTEGER AA(0:40), ROW ,COST
      INTEGER B(40,40),NO(40,40)
      REAL  MINROW, CBS, MINM ,PGS(40), PGF
      REAL P(40,0:40),OP(40,0:40),RR(40,40)
      REAL P1(40), P2(40), CII(40)
      REAL OP1(40),OP2(40),OCII(40)
      REAL R(40), A(40), CFR(40), CI(40), SEQ(40)
      REAL SEK(40)
      REAL MH(40),H(40)
      OPEN(17,FILE='MODEL-2 DATA A')
C
C ********************************************************************
C *****************      READING INPUT DATA     ***********************
C ********************************************************************
C
      READ*, M,N
      READ*,(P(I,0),I=1,N)
      READ*,(P1(I),I=1,N)
      READ*,(P2(I),I=1,N)
      READ*,(CII(I),I=1,N)
      READ*,CFAI
      READ*,CFRI
      READ*,(H(I),I=1,N)
C ********************************************************************
C ******************     INITILIZATION             *****************
C ********************************************************************
C
         DO 798 I=1,N
           OP(I,0) = P(I,0)
           OP1(I)  = P1(I)
           OP2(I)  = P2(I)
           OCII(I) = CII(I)
 798     CONTINUE
C
      DO 700 I = 1, N
```

```
      AA(I) = 0
      DO 701 J = 1, N
701     B(I,J) = 0
700   B(I,I) = 1
C
C ********************************************************************
C ******************       NO INSPECTION (JJ=0)       ******************
C ********************************************************************
C
      JJ = 0
      PG = 1
      DO 10 I = 1, N
          SEQ(I) = I
          PG = PG * (1 - P(I,0))
          P(I,1)=P(I,0)
 10   CONTINUE
C
C ********************************************************************
C ******************       COST WHEN JJ=0              ******************
C ********************************************************************
C
      CBS = CFAI* (1-PG)
      MINM = CBS
      COST = CBS
      WRITE(17,*)'================================================='
      WRITE(17,*)'                                                 '
      WRITE(17,*)'  PG ( 0 ) = ',PG
      WRITE(17,151)CBS
151   FORMAT(2X,'NO INSPECTION COST (JJ=0) = ',F14.3)
      WRITE(17,*)'                                                 '
      WRITE(17,*)'================================================='
      WRITE(17,*)'                                                 '
      WRITE(17,*)'                                                 '
      WRITE(17,*)'                                                 '
      WRITE(17,*)'*****     STARTING THE INSPECTION PROCESS     *****'
      WRITE(17,*)'        ***********************************      '
      WRITE(17,*)'                                                 '
C
C ********************************************************************
C ******************       COST WHEN JJ > 0            ******************
C ********************************************************************
C
 20   JJ = JJ + 1
      DO 911    I = 1,N
911   NO(JJ,I)=B(JJ,I)
       DO 71 I=1,N
       SEQ(I) = I
       P(I,0)=OP(I,0)
       P(I,1)=P(I,0)
       P1(I) =OP1(I)
       P2(I) =OP2(I)
 71    CII(I)=OCII(I)
C---------------------------------------------
      WRITE(17,*)' '
      WRITE(17,*)' '
```

```
      WRITE(17,*)'# OF REPEAT INSP. FOR EACH CHARACT.'
      WRITE(17,*)'------------------------------------ '
      WRITE(17,*)'                                                  '
      WRITE(17,18)(NO(JJ,I),I=1,N)
 18   FORMAT(8X,12(I2,4X))
      WRITE(17,*)'                                                  '
C
C ********************************************************************
C *******************    UPDATING THE P (I,JJ)        *****************
C ********************************************************************
C
       MXM = B(JJ,1)
       DO 28 I=2,N
       IF ( B(JJ,I) .GT. MXM) MXM = B(JJ,I)
 28    CONTINUE
      DO 22 I = 1, N
      DO 33 L = 1,MXM+1
        S1 = P(I,0) * P2(I)**(L-1)
        S2 = (1-P(I,0)) * (1-P1(I))**(L-1)
        P(I,L) = S1/(S1 + S2)
 33   CONTINUE
 22   CONTINUE
C
C ********************************************************************
C *******************    THE SEQUENCING RULE          *****************
C ********************************************************************
C
      DO 48 I=1,N
      DO 48 J=1,MXM
 48   RR(I,J)=(P(I,J)*(1-P2(I)))+((1-P(I,J))*P1(I))
      DO 30 I = 1, N
        F1 = 1
        DO 40 J = 2 ,B(JJ,I)
          S - 1
          DO 50 K = 2, J
 50         S = S * (1-RR(I,K-1))
 40       F1 = F1 + S
        F2  = 1
        DO 60 J = 1, B(JJ,I)
 60       F2 = F2 * ( 1 - RR(I,J))
        IF (B(JJ,I) .EQ. 0) F2 = 1-(P(I,J)*.001)
 30     R(I) = (CII(I)* F1 )/(1-F2)
        DO 66 I = 1, N
 66     SEQ(I)=SEQ(I)
        DO 70 I = 1, N
        DO 70 K = 1, N-1
          IF (R(K) .GT. R(K+1) ) THEN
        CALL SWAP(R(K),R(K+1))
        CALL SWAP(P1(K),P1(K+1))
        CALL SWAP(P2(K),P2(K+1))
        CALL SWAP(CII(K),CII(K+1))
        CALL SWAP(SEQ(K),SEQ(K+1))
        CALL SWAP1(B(JJ,K),B(JJ,K+1))
         DO 44 J= 0,MXM+1
 44     CALL SWAP(P(K,J),P(K+1,J))
```

```
      ENDIF
 70   CONTINUE
      WRITE(17,*) '-------------------------------------------------'
      WRITE(17,*)' THE SEQ. OF THE CHAR. :'
      WRITE(17,*) '-------------------------------------------------'
      WRITE(17,77)(INT(SEQ(I)),I=1,N)
 77   FORMAT(5X,I3)
      WRITE(17,*) '-------------------------------------------------'
C
C *******************************************************************
C ******************     HOW TO CALCULATE A(N)      *****************
C *******************************************************************
C
          S2=1
          DO 238 I=1,N
          S3=(1-P1(I))**B(JJ,I)
  238     S2=S2 * S3
          S6=S2 * PG
C------------------------- IN CASE IF WE HAVE 0 INSPECTIONS -----------
          J=0
  133     J=J+1
          IF (J .EQ. N+1) GO TO 134
          IF (B(JJ,J) .GT. 0) GO TO 133
          NN=J-1
  134     NN=J-1
C---------------------------------------------------------------------
          PGN = 1
          DO 235 I = 1, NN-1
  235       PGN = PGN * ( 1 - P(I,B(JJ,I)+1))
          DO 237 I = NN+1,N
  237       PGN = PGN * ( 1 - P(I,B(JJ,I)))
          PGN = PGN * ( 1 - P(NN,B(JJ,NN)))
          S1=P(NN,B(JJ,NN))*P2(NN)+(1-PGN-P(NN,B(JJ,NN)))*(1-P1(NN))
              S2 = 1
          DO 221 K = 1, NN-1
              S3 = 1
            DO 222 J = 1, B(JJ,K)
              S4 = P(K,J) * P2(K)
              S5 = (1-P(K,J)) * (1 - P1(K))
  222         S3 = S3 * ( S4 + S5)
  221     S2 = S2 * S3
              S3 = 1
              DO 233 J = 1, B(JJ,NN)-1
                S4 =  P(NN,J) * P2(NN)
                S5 = (1-P(NN,J)) * (1-P1(NN))
  233     S3 = S3 * (S4 + S5)
      A(N) = M * (S1 * S2 * S3 + S6 )
C
C *******************************************************************
C ******************     HOW TO CALCULATE CFR       *****************
C *******************************************************************
C
        DO 100 I = 1, N
          S1 = 1
          DO 101 K = 1, I-1
```

```
 101        S1 = S1 * (1-P1(K))**B(JJ,I)
          S2 = 0
          DO 102 K = 1, B(JJ,I)
 102        S2 = S2 + ( 1 - P1(I))**(K-1)
 100      CFR(I) = S1 * S2 * P1(I) * PG * M * CFRI
C
C ********************************************************************
C ******************     HOW TO CALCULATE CFA          ****************
C ********************************************************************
C
C
C------------------------ IN CASE IF WE HAVE 0 INSPECTIONS  ----------
          J=0
 533      J=J+1
          IF (J .EQ. N+1) GO TO 534
          IF (B(JJ,J) .GT. 0) GO TO 533
          NN=J-1
 534      NN=J-1
C---------------------------------------------------------------------
          PGN = 1
          DO 200 I = 1, NN-1
 200        PGN = PGN * ( 1 - P(I,B(JJ,I)+1))
          DO 207 I = NN+1,N
 207        PGN = PGN * ( 1 - P(I,B(JJ,I)))
          PGN = PGN * ( 1 - P(NN,B(JJ,NN)))
        S1=P(NN,B(JJ,NN))*P2(NN)+(1-PGN-P(NN,B(JJ,NN)))*(1-P1(NN))
              S2 = 1
          DO 201 K = 1, NN-1
              S3 = 1
            DO 202 J = 1, B(JJ,K)
              S4 = P(K,J) * P2(K)
              S5 = (1-P(K,J)) * (1 - P1(K))
 202          S3 = S3 * ( S4 + S5)
 201      S2 = S2 * S3
              S3 = 1
              DO 203 J = 1, B(JJ,NN)-1
                S4 =  P(NN,J) * P2(NN)
                S5 = (1-P(NN,J)) * (1-P1(NN))
 203      S3 = S3 * (S4 + S5)
      CFAF = S1 * S2 * S3   * M * CFAI
           IF(CFAF .LT. 0.0)THEN
           CFAF=0.0
           ENDIF
C
C ********************************************************************
C ******************     HOW TO CALCULATE CI(I)        ****************
C ********************************************************************
C
C
      DO 300 I = 1, N
          S1=0.0
          DO 310 K=1,B(JJ,I)
          S2 = 1
          DO 320 J = 1, K-1
            S4 = P(I,J) * P2(I)
```

```
           S5 = (1-P(I,J)) * (1-P1(I))
 320       S2 = S2 * (S4 + S5)
 310       S1 = S1+S2
       S2=1
      DO 400 L = 1,I-1
       S3=1
       DO 410 J = 1, B(JJ,L)
         S4 = P(L,J) * P2(L)
         S5 = (1-P(L,J)) * (1-P1(L))
 410     S3 = S3 * (S4 + S5)
 400   S2 = S2 * S3
 300   CI(I) =  CII(I) * M  *  S1  * S2
C
C ********************************************************************
C ******************     HOW TO CALCULATE MH(I)      *****************
C ********************************************************************
C
      DO 337 I = 1, N
          S1=1.0
       IF ( B(JJ,I) .EQ. 0.0) MH(I)=0.0
          DO 311 K=1,B(JJ,I)
          DO 323 J = 1, K-1
            S4 = P(I,J) * P2(I)
            S5 = (1-P(I,J)) * (1-P1(I))
 323        S1 = S1 * (S4 + S5)
        S2=1
      DO 408 L = 1,I-1
        S3=1
        DO 413 J = 1, B(JJ,L)
          S4 = P(L,J) * P2(L)
          S5 = (1-P(L,J)) * (1-P1(L))
 413      S3 = S3 * (S4 + S5)
 408   S2 = S2 * S3
       IF ( K .EQ. 1 )THEN
       MH(I) = H(I) * CII(I) * M * S2
       ELSE
       MH(I) = ( .5 * H(I) * CII(I) * M * S1 * S2 ) + MH(I)
       ENDIF
 311   CONTINUE
 337   CONTINUE
C
C ********************************************************************
C ******************     HOW TO CALCULATE TCFR      ******************
C ********************************************************************
C
        TCFR = 0
        DO 500 I = 1, N
 500      TCFR = TCFR + CFR(I)
        WRITE(17,2)TCFR
  2       FORMAT(2X,'T C F R       = ',F14.3)
C
C ********************************************************************
C ******************     HOW TO CALCULATE TCFA      ******************
C ********************************************************************
C
```

```
      TCFA = CFAF
      WRITE(17,3)TCFA
  3      FORMAT(2X,'T C F A       = ',F14.3)
C
C *****************************************************************
C ******************   HOW TO CALCULATE TCI        ******************
C *****************************************************************
C
      TCI = 0
      DO 600 I = 1, N
 600     TCI = TCI + CI(I)
      WRITE(17,4)TCI
  4      FORMAT(2X,'T C I         = ',F14.3)
C
C *****************************************************************
C ******************   HOW TO CALCULATE TCMH       ******************
C *****************************************************************
C
      TCMH = 0
      DO 615 I = 1, N
 615     TCMH = TCMH + MH(I)
      WRITE(17,81)TCMH
 81       FORMAT(2X,'T C M H       = ',F14.3)
C
C *****************************************************************
C ******************   HOW TO CALCULATE TA         ******************
C *****************************************************************
C
      TA = A(N)
      WRITE(17,5)A(N)
  5      FORMAT(2X,'T A           = ',F14.3)
C
C *****************************************************************
C ******************   HOW TO CALCULATE ETC(JJ)    ******************
C *****************************************************************
C
      MINROW = ( TCFR + TCFA + TCI + TCMH )/TA
      WRITE(17,6)MINROW
  6      FORMAT(2X,'E T C         = ',F14.3)
C
C *****************************************************************
C ******************    PG AFTER INSPECTION        ******************
C *****************************************************************
C
           PGS(JJ)=1
         DO 488 I  = 1, NN
 488        PGS(JJ) = PGS(JJ) * ( 1 - P(I,B(JJ,I)+1))
         DO 489 I = NN+1,N
 489        PGS(JJ) = PGS(JJ) * ( 1 - P(I,B(JJ,I)))
      WRITE(17,269)PGS(JJ)
 269  FORMAT(2X,'PG AFTER INSP.= ',6X,F14.9)
      WRITE(17,*) '-----------------------------------------------'
C
C *****************************************************************
C ******************   STOPPING CRITERION          ******************
```

```
C *********************************************************************
C
      IF ( MINROW .LT. MINM ) THEN
        ROW = JJ
        MINM  = MINROW
        DO 779 I=1,N
779     SFK(I)=SEQ(I)
      ENDIF
      IF ( JJ .LT. N) GOTO 20
      IF (MINM .LT. CBS) THEN
        DO 777 I = 1, N
777       AA(I) = NO(ROW,I)
        CBS = MINM
        PGF = PGS(ROW)
        CALL CHANGE(NO,B,ROW,N)
        JJ = 0
        GO TO 20
      ENDIF
C
C *********************************************************************
C ******************    PRINTING THE RESULTS       ******************
C *********************************************************************
C
      WRITE(17,*)'  '
      WRITE(17,*)'  '
      WRITE(17,*)'          *    *    *    *   *     '
      WRITE(17,*)'  '
      WRITE(17,*)'  '
      WRITE(17,*)'THE OPTIMAL SOLUTION FOR THIS PROBLEM  :  '
      WRITE(17,*)'==========================================='
      WRITE(17,*)'  '
      WRITE(17,8)(AA(I),I=1,N)
 8    FORMAT(8X,12(I2,4X))
      WRITE(17,*)'  '
      WRITE(17,*)'==========================================='
      WRITE(17,*)' '
      WRITE(17,9)CBS
 9    FORMAT(2X,'OPTIMAL COST   =',F14.3)
      WRITE(17,117)PGF
 117  FORMAT(2X,'PG             =',F14.9)
      WRITE(17,*)' '
      WRITE(17,*)'==========================================='
      WRITE(17,*)'  '
      WRITE(17,*)'  OPTIMAL SEQ.    '
      WRITE(17,*)'  ---------------'
      WRITE(17,7)(INT(SFK(I)),I=1,N)
 7    FORMAT(5X,I3)
      WRITE(17,*)'   '
      WRITE(17,*)'==========================================='
      WRITE(17,*)'   '
      WRITE(17,*)'   '
      WRITE(17,*)'--------------NO ENTERIES BEYOND THIS LINE -------'
      WRITE(17,*)'   '
      WRITE(17,*)'  '
      WRITE(17,*)'  '
```

```
      WRITE(17,*)'  '
      WRITE(17,*)'  '
      WRITE(17,*)'  '
      WRITE(17,*)'  '
      WRITE(17,*)'  '
      CLOSE(17)
      END
C
C***********************************************************************
C*******  THIS SUBROUTINE IS TO SWAP THE CHARACTERISTICS          ******
C*******  WITH THERE ASSOCIATED DATA.                             ******
C***********************************************************************
C

      SUBROUTINE SWAP(A,B)
      T = A
      A = B
      B = T
      END

      SUBROUTINE SWAP1(IA,IB)
      IT = IA
      IA = IB
      IB = IT
      END

      SUBROUTINE CHANGE(NO,B,K,N)
      INTEGER A(40), B(40,40)
      INTEGER NO(40,40)
      DO 10 I = 1, N
 10     A(I) = NO(K,I)
      DO 20 I = 1, N
      DO 30 J = 1, N
 30     B(I,J) = A(J)
 20   B(I,I) = B(I,I) + 1
      END
C$DATA
100  3
0.1     0.2     0.3
0.01    0.01    0.01
0.015   0.015   0.015
100     100     100
100000
500
0  0  0
```

APPENDIX (C)

```
C
C
C
C
C                      ***************************
C                      ***                       ***
C                      ***     APPENDIX C        ***
C                      ***                       ***
C                      ***************************
C
C
C
C*********************************************************************
C********                                                     ********
C********      A LISTING OF THE PROGRAM USED TO RANDOMLY      ********
C********      GENERATE 100 PROBLEMS.                         ********
C********                                                     ********
C*********************************************************************
C
C
C
      REAL P(40),P1(40),P2(40)
      REAL A,B,U,U1,U2,V1,V2,W,Y
      INTEGER N ,CI(40),CA,CR
      COMMON /IW/IX
C
C
C**********************************************************************
C*********                                                    *********
C*********         J = # OF PROBLEMS TO BE GENARATED.         *********
C*********                                                    *********
C**********************************************************************
C
          DO 1 J=1,100
C
C**********************************************************************
C*********                                                    *********
C*********         IX = THE SEED USED.                        *********
C*********                                                    *********
C**********************************************************************
C
          1X=197+( 30 * J )
C
C**********************************************************************
C*********                                                    *********
C*********         N=NUMBER OF CHARACTERISTICS.               *********
C*********   ASSUMED TO HAVE A UNIFORM DISTRIBUTION           *********
C*********   BETWEEN A NAD B.                                 *********
C*********                                                    *********
C**********************************************************************
C
        A=1.0
        B=10.0
          DO 10 I=1,120
 10         U=RANUN(IX)
```

```
            N = A + ( B * U )
C
C*********************************************************************
C*********                                                   *********
C*********      CI(I)=COST OF INSPECTING CHARACTERISTIC I.   *********
C*********      ASSUMED TO HAVE A UNIFORM DISTRIBUTION       *********
C*********      BETWEEN A NAD B.                             *********
C*********                                                   *********
C*********************************************************************
C
C
              DO 20 I=1,N
              A=10.0
              B=100.0
         CI(I) = UNIFRM(A,B)
 20      CONTINUE
C
C
C*********************************************************************
C*********                                                   *********
C*********      CA =COST OF FALSE ACCEPTANCE.                *********
C*********      ASSUMED TO HAVE A UNIFORM DISTRIBUTION       *********
C*********      BETWEEN A NAD B.                             *********
C*********                                                   *********
C*********************************************************************
C
              A=100000.0
              B=1000000.0
         CA = UNIFRM(A,B)
C
C*********************************************************************
C*********                                                   *********
C*********      CR =COST OF FALSE REJECTION.                 *********
C*********      ASSUMED TO HAVE A UNIFORM DISTRIBUTION       *********
C*********      BETWEEN A NAD B.                             *********
C*********                                                   *********
C*********************************************************************
C
              A = 500.0
              B = 1000.0
         CR = UNIFRM(A,B)
C
C*********************************************************************
C*********                                                   *********
C*********      P(I)= PROBABILITY OF CHARACTERISTIC I BEING  *********
C*********            DEFFECTIVE.                            *********
C*********      ASSUMED TO HAVE A NORMAL DISTRIBUTION        *********
C*********      WITH MEAN=0.15 AND VARIANCE=(0.048)**2       *********
C*********                                                   *********
C*********************************************************************
C
         DO 40 I=1,N
 44      A=0.0
         B=1.0
         U1=UNIFRM(A,B)
```

```
      U2=UNIFRM(A,B)
      V1=2*U1-1
      V2=2*U2-1
      W= V1**2 + V2**2
         IF (W .GT. 1.0) GO TO 44
      Y=((-2 * LOG(W))/W)**0.5
      X1=V1*Y
      X2=V2*Y
      IF ( MOD(J,2) .EQ. 0.0) THEN
      P(I)=.15 + .048 * X2
      ELSE
      P(I)=.15 + .048 * X1
      ENDIF
 40   CONTINUE
C
C*****************************************************************
C*********                                              *********
C*********     P1(I)= PROBABILITY OF TYPE I ERROR.      *********
C*********     ASSUMED TO HAVE A NORMAL DISTRIBUTION    *********
C*********     WITH MEAN=0.1  AND VARIANCE=(0.03)**2    *********
C*********                                              *********
C*****************************************************************
C
      DO 50 I=1,N
 55   A=0.0
      B=1.0
      U1=UNIFRM(A,B)
      U2=UNIFRM(A,B)
      V1=2*U1-1
      V2=2*U2-1
      W= V1**2 + V2**2
         IF (W .GT. 1.0) GO TO 55
      Y=((-2 * LOG(W))/W)**0.5
      X1=V1*Y
      X2=V2*Y
      IF ( MOD(J,2) .EQ. 0.0) THEN
      P1(I)=.10 + .03 * X2
      ELSE
      P1(I)=.10 + .03 * X1
      ENDIF
 50   CONTINUE
C
C*****************************************************************
C*********                                              *********
C*********     P2(I)= PROBABILITY OF TYPE II ERROR.     *********
C*********     ASSUMED TO HAVE A NORMAL DISTRIBUTION    *********
C*********     WITH MEAN=0.1  AND VARIANCE=(0.03)**2    *********
C*********                                              *********
C*****************************************************************
C
      DO 60 I=1,N
 66   A=0.0
      B=1.0
      U1=UNIFRM(A,B)
      U2=UNIFRM(A,B)
```

```
      V1=2*U1-1
      V2=2*U2-1
      W= V1**2 + V2**2
         IF (W .GT. 1.0) GO TO 66
      Y=((-2 * LOG(W))/W)**0.5
      X1=V1*Y
      X2=V2*Y
      IF ( MOD(J,2) .EQ. 0.0) THEN
      P2(I)=.10 + .03 * X2
      ELSE
      P2(I)=.10 + .03 * X1
      ENDIF
 60   CONTINUE
C
C**************************************************************************
C*********                                                        *********
C*********          PRINTING THE RESULTS.                         *********
C*********                                                        *********
C**************************************************************************
C
C
      PRINT98,J
 98   FORMAT(14X,'PROBLEM # (',2X,I3,4X,')')
      PRINT*,'                ','-----------------------'
      PRINT11,N
 11   FORMAT(I2)
      PRINT41,(P(I),I=1,N)
 41   FORMAT(10F7.3)
      PRINT51,(P1(I),I=1,N)
 51   FORMAT(10F7.3)
      PRINT61,(P2(I),I=1,N)
 61   FORMAT(10F7.3)
      PRINT22,(CI(I),I=1,N)
 22   FORMAT(10I7)
      PRINT23,CA
 23   FORMAT(I7)
      PRINT24,CR
 24   FORMAT(I4)

     PRINT*,'---------------------------------------------------------'


 1    CONTINUE
      END
C
C
      FUNCTION UNIFRM(A,B)
      REAL A,B,U
      COMMON /IW/IX
      U=RANUN(IX)
      UNIFRM=A+(U*(B-A))
      RETURN
      END
C
C
```

123C

```
      FUNCTION RANUN(IX)
      IX=MOD(25173*IX+13849,65536)
      RANUN = REAL(IX)/65536
      RETURN
      END
```

APPENDIX (D)

Models Comparison

The following table summarizes the results of solving one hundred randomly generated inspection problems. The comparison is based on the expected total inspection cost.

Legend:

R	**: Raouf et al Model (1983)**
M1	**: Model (1) in the thesis**
M2	**: Model (2), the general model, in the thesis**
E.T.C.	**: Expected Total Cost**
PG	**: The probability of a component being good**
R.I.	**: Repeat Inspections**

Prob. #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
1	R	10	16837.68	0.97479	2	3,7,4,9,5,10,2,8,6,1
	M1		16550.56	0.97479	2	3,7,10,4,5,9,2,8,6,1
	M2		16550.56	0.97479	2,2,2,2,2,2,2,2,2,2	3,7,10,4,5,9,2,8,6,1
2	R	8	14369.25	0.99755	3	3,8,6,2,5,4,7,1
	M1		15184.17	0.99755	3	8,3,2,5,6,7,4,1
	M2		12772.22	0.99324	3,2,2,2,3,3,2,3	3,2,8,5,6,7,4,1
3	R	6	11461.96	0.99790	3	4,5,3,2,1,6
	M1		11125.82	0.99790	3	5,4,3,2,6,1
	M2		9651.44	0.99534	2,2,3,2,4,3	4,5,3,2,1,6
4	R	3	2714.05	0.99550	3	2,3,1
	M1		2708.58	0.99550	3	2,3,1
	M2		2708.58	0.99550	3,3,3	2,3,1
5	R	1	251.51	0.99996	3	1
	M1		251.51	0.99996	3	1
	M2		251.51	0.99996	3	1

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
6	R	9	22178.27	0.98555	2	8,2,7,4,3,,1,9,6,5
	M1		21959.75	0.98555	2	8,2,7,4,3,,1,6,5,9
	M2		18619.05	0.99569	3,3,3,2,2,2,2,3,3	8,2,7,4,3,6,1,5,9
7	R	6	9881.69	0.99714	3	6,4,5,2,3,1
	M1		9961.51	0.99714	3	4,6,5,2,3,1
	M2		8617.31	0.99464	3,2,3,3,3,2	4,6,5,2,3,1
8	R	4	3674.51	0.99171	2	1,4,3,2
	M1		3686.05	0.99171	2	1,4,3,2
	M2		3412.63	0.99565	2,2,3,2	1,4,3,2
9	R	2	1831.15	0.99971	3	2,1
	M1		1850.37	0.99971	3	2,1
	M2		1651.03	0.99956	2,2	2,1
10	R	9	27291.75	0.98057	2	9,6,1,4,8,7,2,3,5
	M1		26743.64	0.99776	3	9,6,1,4,4,5,8,7,2
	M2		20867.09	0.99285	3,3,2,2,2,3,2,2,3	9,6,1,4,4,5,8,7,2
11	R	7	13568.44	0.99763	3	7,3,4,2,1,6,5
	M1		13141.86	0.99763	3	7,3,4,2,1,5,6
	M2		11651.67	0.99585	3,2,2,3,3,3,3	7,3,4,2,1,5,6
12	R	5	5827.14	0.99118	2	4,3,5,1,2
	M1		5740.46	0.99118	2	4,3,5,1,2
	M2		4858.68	0.99636	1,3,2,3,3	4,3,5,1,2
13	R	2	1369.20	0.99931	3	1,2
	M1		1370.27	0.99931	3	1,2
	M2		1215.86	0.99887	3,2	1,2
14	R	10	32203.05	0.98062	2	9,4,1,6,8,7,2,10,2,5
	M1		32959.37	0.98062	2	9,4,6,1,8,3,7,10,5,2
	M2		30684.31	0.98652	2,2,2,2,3,2,2,2,2,3	9,4,6,1,8,3,7,2,10,5
15	R	8	27880.02	0.96944	2	6,2,8,3,7,,5,1,4
	M1		27463.20	0.96944	2	6,2,8,3,7,,5,1,4
	M2		24781.41	0.98262	2,2,2,1,3,2,3,2	6,2,8,3,7,,5,1,4
16	R	5	7331.69	0.99812	3	1,4,3,2,5
	M1		7044.89	0.99812	3	1,4,3,2,5
	M2		6819.41	0.99755	3,3,3,2,3	1,4,3,2,5
17	R	3	1950.36	0.99922	3	2,3,1
	M1		1937.62	0.99922	3	2,3,1
	M2		1609.75	0.99968	4,3,3	2,3,1

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
18	R	1	518.24	0.99975	2	1
	M1		518.24	0.99975	2	1
	M2		518.24	0.99975	2	1
19	R	9	15467.73	0.98594	2	2,1,8,6,4,7,9,5,3
	M1		15444.74	0.98594	2	2,1,8,7,9,6,4,3,5
	M2		13856.67	0.99495	3,2,3,2,2,2,3,2,2	2,1,8,9,6,7,4,3,5
20	R	6	8522.97	0.99655	3	6,4,2,1,3,5
	M1		8441.02	0.99655	3	6,4,2,1,3,5
	M2		8130.52	0.99741	3,3,3,3,4,3	6,4,2,1,3,5
21	R	4	4312.86	0.99889	3	3,2,4,1
	M1		4329.22	0.99889	3	3,2,1,4
	M2		4329.22	0.99889	3,3,3,3	3,2,1,4
22	R	2	1295.55	0.99959	3	2,1
	M1		1299.79	0.99959	3	2,1
	M2		1299.79	0.99959	3,3	2,1
23	R	9	21207.33	0.97887	2	5,7,3,9,4,1,8,6,2
	M1		21190.68	0.97887	2	5,7,3,4,9,1,8,6,2
	M2		20086.30	0.98395	2,2,1,2,3,3,2,2,2	5,7,3,4,9,1,6,8,2
24	R	7	21903.80	0.99680	3	2,3,7,4,6,5,1
	M1		22537.01	0.99680	3	2,3,7,4,6,1,5
	M2		17849.17	0.99218	3,2,2,3,3,2,3	3,2,7,4,6,1,5
25	R	5	4695.13	0.98893	2	4,1,3,5,2
	M1		4664.77	0.98893	2	4,1,3,5,2
	M2		4664.77	0.98893	2,2,2,2,2	4,1,3,5,2
26	R	2	2378.24	0.99862	3	2,1
	M1		2381.86	0.99862	3	2,1
	M2		2310.48	0.99936	3,4	2,1
27	R	10	30330.89	0.97387	2	6,10,4,3,9,5,7,8,1,2
	M1		30814.30	0.97387	2	6,10,4,9,3,5,7,2,8,1
	M2		28418.63	0.98402	2,2,3,4,2,3,2,2,2,2	6,10,4,3,9,5,7,2,8,1
28	R	8	9716.10	0.98382	2	7,5,2,4,8,1,3,6
	M1		9693.19	0.98382	2	7,5,2,4,1,8,3,6
	M2		9072.81	0.98780	2,3,2,2,2,2,2,2	7,5,2,4,1,8,3,6
29	R	5	7372.77	0.99109	2	5,4,1,2,3
	M1		7321.37	0.99902	3	5,4,1,2,3
	M2		6256.01	0.99704	2,3,3,3,2	5,4,1,2,3

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
30	R	3	4883.51	0.99942	3	1,2,3
	M1		4717.34	0.99942	3	1,2,3
	M2		4423.22	0.99876	2,3,3	1,2,3
31	R	1	740.66	0.99994	4	1
	M1		740.66	0.99994	4	1
	M2		740.66	0.99994	4	1
32	R	9	23293.12	0.99738	3	6,8,3,5,9,7,2,1,4
	M1		23764.44	0.99738	3	6,8,3,5,9,7,2,1,4
	M2		20795.04	0.99421	2,3,2,2,3,3,3,2,3	6,8,3,5,9,7,2,1,4
33	R	6	10041.86	0.99876	3	1,3,4,2,5,6
	M1		9740.18	0.99876	3	1,3,4,2,5,6
	M2		9409.26	0.99816	3,3,3,3,3,2	1,3,4,2,5,6
34	R	4	2801.04	0.99329	2	3,2,1,4
	M1		2812.45	0.99329	2	3,1,2,4
	M2		2753.60	0.99608	2,2,3,2	3,1,2,4
35	R	2	1163.76	0.99979	3	2,1
	M1		1146.43	0.99979	3	2,1
	M2		1146.43	0.99979	23,3	2,1
36	R	9	35473.30	0.97365	2	1,8,2,9,7,6,4,3,5
	M1		35564.20	0.97365	2	1,8,9,3,7,6,2,4,5
	M2		29178.91	0.98810	2,2,2,2,3,3,3,2,3	1,8,3,9,2,4,7,6,5
37	R	7	11047.61	0.99726	3	6,2,7,5,3,1,4
	M1		10870.15	0.99726	3	6,2,1,5,7,3,4
	M2		10243.54	0.99746	4,3,3,3,2,3,3	6,2,5,1,7,3,4
38	R	5	5665.39	0.99893	3	4,5,3,2,1
	M1		5299.45	0.99893	3	4,5,3,2,1
	M2		5299.45	0.99893	3,3,3,3,3	4,5,3,2,1
39	R	2	2114.10	0.99945	3	1,2
	M1		2122.63	0.99945	3	1,2
	M2		2094.08	0.99983	3,4	1,2
40	R	10	37667.43	0.96891	2	7,1,10,9,2,4,8,5,6,3
	M1		38625.18	0.96891	2	7,1,9,10,2,5,4,8,6,3
	M2		36963.65	0.96243	2,2,2,2,2,2,1,2,2,2	7,1,9,10,2,5,4,8,6,3
41	R	8	19823.91	0.99788	3	3,2,4,7,8,5,1,6
	M1		20358.21	0.99788	3	3,2,4,7,6,8,5,1
	M2		17786.48	0.99518	2,3,3,3,2,3,3,2	3,2,4,7,8,6,5,1

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
42	R	5	5351.00	0.99060	2	4,1,3,2,5
	M1		5342.09	0.99060	2	4,1,3,5,2
	M2		5105.86	0.99389	2,2,3,2,3	4,1,3,2,5
43	R	3	2732.79	0.99917	3	2,3,1
	M1		2707.61	0.99917	3	2,3,1
	M2		2653.81	0.99960	4,3,3	2,3,1
44	R	1	843.14	0.99968	3	1
	M1		843.14	0.99968	3	1
	M2		843.14	0.99968	3	1
45	R	9	28544.11	0.96476	2	9,7,1,4,8,3,6,2,5
	M1		28982.04	0.96476	2	7,9,1,3,8,4,6,2,5
	M2		25673.47	0.97999	3,2,3,2,3,2,2,2,2	7,9,1,8,4,3,6,2,5
46	R	6	8855.40	0.99798	3	4,2,3,5,1,6
	M1		8825.58	0.99798	3	4,2,3,5,6,1
	M2		7288.26	0.99455	2,3,2,2,3,3	4,2,3,5,1,6
47	R	4	4887.59	0.99933	3	4,2,1,3
	M1		4987.58	0.99933	3	4,2,1,3
	M2		4450.87	0.99867	2,3,3,3	4,2,1,3
48	R	2	1689.54	0.99989	4	1,2
	M1		1723.51	0.99989	4	1,2
	M2		1589.04	0.99963	3,4	1,2
49	R	9	13132.36	0.98723	2	5,9,2,8,4,6,3,7,1
	M1		13279.46	0.98723	2	5,9,2,8,4,6,1,3,7
	M2		13024.80	0.99073	2,2,2,2,3,2,2,2,2,	5,9,2,8,4,6,1,3,7
50	R	7	16968.27	0.99590	3	7,1,4,2,6,3,5
	M1		15835.87	0.99590	3	7,1,4,2,6,3,5
	M2		14795.27	0.99684	3,4,4,2,3,3,3	7,1,4,2,6,3,5
51	R	5	6466.48	0.99825	3	3,4,1,2,5
	M1		6134.46	0.99825	3	3,4,1,2,5
	M2		6134.46	0.99825	3,3,3,3,3	3,4,1,2,5
52	R	2	1599.54	0.99956	3	2,1
	M1		1566.70	0.99956	3	2,1
	M2		1520.80	0.99825	3,2	2,1
53	R	10	37397.96	0.99530	3	1,8,10,9,3,4,2,5,6,7
	M1		35545.39	0.99530	3	1,8,10,3,9,5,4,7,6,2
	M2		28586.68	0.98996	3,3,3,2,3,2,3,3,2,2	1,8,10,3,9,4,5,6,7,2

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
54	R	8	20719.65	0.98353	2	2,5,6,4,3,1,7,8
	M1		20972.38	0.98353	2	2,5,6,3,4,7,1,8
	M2		20000.16	0.98983	3,2,3,2,2,3,2,2	2,5,6,3,4,7,1,8
55	R	5	5907.77	0.99856	3	5,4,2,1,3
	M1		5793.42	0.99856	3	5,4,2,1,3
	M2		5746.41	0.99880	3,3,4,3,3	5,4,2,1,3
56	R	3	3522.10	0.99964	3	1,3,2
	M1		3419.69	0.99964	3	1,3,2
	M2		3419.69	0.99964	3,3,3	1,3,2
57	R	1	657.47	0.99991	4	1
	M1		657.47	0.99991	4	1
	M2		657.47	0.99991	4	1
58	R	8	26361.38	0.98579	2	5,8,7,2,1,6,4,3
	M1		27032.29	0.98579	2	5,7,8,1,2,4,3,6
	M2		26210.73	0.99128	2,2,3,2,2,2,2,3	5,7,8,1,2,4,6,3
59	R	6	7826.23	0.99835	3	1,4,2,6,5,3
	M1		7766.39	0.99835	3	1,4,2,6,5,3
	M2		6677.96	0.99730	2,3,2,3,3,3	1,4,2,6,5,3
60	R	4	3312.53	0.99359	2	4,1,3,2
	M1		3255.87	0.99359	2	4,1,3,2
	M2		3147.71	0.99756	2,3,3,2	4,1,3,2
61	R	2	1283.97	0.99738	2	2,1
	M1		1290.15	0.99738	2	2,1
	M2		1290.15	0.99738	2,2	2,1
62	R	9	34664.35	0.97103	2	9,6,3,7,1,8,5,4,2
	M1		34683.30	0.97103	2	9,6,7,3,1,8,5,4,2
	M2		26066.15	0.98862	2,2,2,3,2,2,3,3,3	9,6,3,7,1,8,5,4,2
63	R	7	6815.79	0.98882	2	2,3,7,4,6,1,5
	M1		7031.57	0.98882	2	2,7,3,4,5,1,6
	M2		7031.57	0.98882	2,2,2,2,2,2,2	2,7,3,4,5,1,6
64	R	5	5296.36	0.99902	3	4,5,2,3,1
	M1		5226.54	0.99902	3	4,5,3,2,1
	M2		4537.36	0.99791	2,3,3,3,2	4,5,3,1,2
65	R	2	1570.70	0.99940	3	2,1
	M1		1581.00	0.99940	3	2,1
	M2		1581.00	0.99940	3,3	2,1

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
66	R	10	37332.84	0.97109	2	8,9,2,1,10,5,4,3,6,7
	M1		38185.09	0.97109	2	8,9,2,10,1,5,4.6,3,7
	M2		36815.08	0.98165	3,2,2,2,2,3,2,3,2,2	8,9,2,10,5,4,1,3,7,6
67	R	8	16076.10	0.99739	3	2,7,6,4,1,3,5,8
	M1		14680.09	0.99739	3	2,7,6,4,3,1,5,8
	M2		12425.11	0.99366	2,3,3,3,3,2,2,3	2,7,6,4,1,3,5,8
68	R	5	7578.40	0.99832	3	4,1,3,2,5
	M1		7758.22	0.99832	3	4,1,2,5,3
	M2		7684.97	0.99698	3,3,3,2,3	4,1,2,5,3
69	R	3	3522.05	0.99875	3	3,1,2
	M1		3558.97	0.99875	3	3,2,1
	M2		3534.21	0.99913	3,4,3	3,2,1
70	R	1	516.69	0.99983	3	1
	M1		516.69	0.99983	3	1
	M2		516.69	0.99983	3	1
71	R	8	20610.42	0.98364	2	3,6,1,8,4,7,5,2
	M1		20716.02	0.98364	2	3,1,6,8,4,7,5,2
	M2		1979.00	0.98977	2,3,2,2,2,2,3,2	3,1,6,8,4,7,5,2
72	R	6	10026.12	0.99775	3	6,3,2,4,1,5
	M1		9740.75	0.99775	3	6,3,2,4,1,5
	M2		9740.75	0.99775	3,3,3,3,3,3	6,3,2,4,1,5
73	R	4	4072.54	0.99860	3	1,4,3,2
	M1		4028.05	0.99860	3	4,1,3,2
	M2		3596.04	0.99894	2,3,3,4	1,4,3,2
74	R	2	1985.22	0.99927	3	1,2
	M1		2006.68	0.99927	3	2,1
	M2		1855.21	0.99856	2,4	1,2
75	R	9	20685.33	0.96833	2	5,8,3,7,9,4,2,1,6
	M1		21149.47	0.96833	2	5,8,2,4,7,3,9,1,6
	M2		19830.01	0.98273	3,3,2,2,2,3,3,2,2	5,8,4,3,9,7,2,1,6
76	R	7	10915.22	0.98046	2	5,2,1,3,4,6,7
	M1		10598.91	0.98046	2	5,2,1,4,6,3,7
	M2		8330.65	0.99455	3,3,2,3,2,3,2	5,2,1,4,6,3,7
77	R	5	5288.47	0.98714	2	4,1,3,5,2
	M1		5306.86	0.98714	2	4,1,2,3,5
	M2		5045.20	0.99281	3,3,2,2,2	4,1,3,2,5

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
78	R	2	1787.93	0.99953	3	2,1
	M1		1760.10	0.99953	3	2,1
	M2		1760.10	0.99953	3,3	2,1
79	R	10	22948.74	0.98742	2	1,10,7,6,2,8,4,3,5,9
	M1		23223.60	0.98742	2	1,7,10,6,8,2,5,4,3,9
	M2		22730.15	0.98189	2,2,2,2,2,2,3,2,2,2	1,10,7,6,8,2,5,4,9,3
80	R	8	16386.52	0.97986	2	8,5,4,7,1,6,3,2
	M1		16274.99	0.97986	2	8,5,7,4,1,6,3,2
	M2		16274.99	0.97986	2,2,2,2,2,2,2,2	8,5,7,4,1,6,3,2
81	R	5	4887.86	0.99829	3	5,3,4,2,1
	M1		4717.54	0.99829	3	5,3,4,2,1
	M2		4576.32	0.99829	3,3,2,4,3	5,3,2,4,1
82	R	3	1561.80	0.99901	3	1,2,3
	M1		1584.81	0.99901	3	1,3,2
	M2		1569.46	0.99654	3,2,3	1,2,3
83	R	1	624.69	0.99976	3	1
	M1		624.69	0.99976	3	1
	M2		624.69	0.99976	3	1
84	R	8	16870.27	0.98410	2	7,3,4,5,6,2,8,1
	M1		16834.95	0.98410	2	7,3,6,5,4,8,2,1
	M2		16485.48	0.98602	3,2,2,2,2,2,2,2	7,3,6,5,4,8,2,1
85	R	6	6278.54	0.99185	2	1,5,4,2,3,6
	M1		6362.10	0.99185	2	1,5,4,2,3,6
	M2		5844.00	0.99607	2,2,3,2,3,2	1,5,4,2,3,6
86	R	4	3043.68	0.99449	2	4,3,2,1
	M1		3044.79	0.99449	2	4,2,3,1
	M2		3044.79	0.99449	2,2,2,2	4,2,3,1
87	R	1	481.43	0.99955	3	1
	M1		481.43	0.99955	3	1
	M2		481.43	0.99955	3	1
88	R	9	42674.39	0.96629	2	1,6,9,4,8,5,2,7,3
	M1		43031.46	0.96629	2	1,6,9,8,4,5,7,3,2
	M2		38869.51	0.97736	2,2,3,1,3,3,3,2,2	1,9,6,4,8,5,7,2,3
89	R	7	12964.84	0.98510	2	2,6,4,3,1,5,7
	M1		12978.39	0.98510	2	2,6,5,4,3,1,7
	M2		11269.13	0.99282	3,2,2,3,3,2,2	2,6,3,4,5,1,7

Prob #	Model Used	# of Char.	E.T.C.	PG	Optimal # of R.I.	Optimal Sequence
90	R	5	5752.44	0.99852	3	1,5,2,4,3
	M1		5520.07	0.99852	3	1,5,4,2,3
	M2		5520.07	0.99852	3,3,3,3,3	1,5,4,2,3
91	R	2	1166.60	0.99647	2	2,1
	M1		1161.70	0.99647	2	2,1
	M2		1093.56	0.99848	3,2	2,1
92	R	10	19523.03	0.99595	2	3,6,2,7,8,10,4,9,1,5
	M1		19447.73	0.99595	2	3,6,2,7,8,10,4,9,1,5
	M2		18364.60	0.99519	2,2,2,2,2,1,2,2,2,2	3,6,2,7,8,10,4,9,1,5
93	R	8	17009.45	0.99747	3	1,8,3,6,4,2,7,5
	M1		16790.47	0.99747	3	1,8,3,4,6,2,5,7
	M2		15553.71	0.99640	3,3,3,2,3,3,2,3	1,8,3,4,6,2,5,7
94	R	5	4894.85	0.99822	3	3,5,4,2,1
	M1		5134.79	0.99822	3	3,4,2,1,5
	M2		4711.16	0.99678	2,2,3,3,3	3,4,2,1,5
95	R	3	2764.09	0.99818	3	3,1,2
	M1		2822.17	0.99818	3	3,1,2
	M2		2603.90	0.99703	3,2,3	3,1,2
96	R	1	329.38	0.99994	4	1
	M1		329.38	0.99994	4	1
	M2		329.38	0.99994	4	1
97	R	8	7396.07	0.98742	2	8,6,1,7,3,2,5,4
	M1		7398.82	0.98742	2	8,6,7,1,3,5,2,4
	M2		7173.93	0.99060	2,2,2,3,2,3,2,2	8,6,7,1,3,5,2,4
98	R	6	8401.74	0.99710	3	2,4,6,3,5,1
	M1		8247.22	0.99710	3	2,4,6,3,5,1
	M2		8044.33	0.99677	3,3,3,2,4,3	2,4,6,3,5,1
99	R	4	5461.21	0.99914	3	4,1,3,2
	M1		5297.78	0.99914	3	4,1,2,3
	M2		5026.53	0.99832	3,3,2,3	4,1,2,3
100	R	1	919.93	0.99943	3	1
	M1		919.93	0.99943	3	1
	M2		919.93	0.99943	3	1

Models Comparison

The following table summarizes the results of solving one hundred randomly generated inspection problems using the two models in the thesis and the one in the literature. The comparison is based on the expected total inspection cost with material handling.

	Expected Total Inspection Cost (with material handling) Using:		
Problem #	Raouf et al Model	Model (1) in the thesis	Model (2) in the thesis
1	17562.41	17029.06	17029.06
2	15101.86	15669.95	13095.50
3	11903.07	11346.80	09817.26
4	02826.52	02771.39	02771.39
5	00260.11	00260.11	00260.11
6	22684.79	22290.48	19128.63
7	10362.50	10241.11	08836.37
8	03810.85	03778.44	03526.06
9	01898.80	01895.14	01688.09
10	27895.50	27114.14	21339.90
11	14198.68	13472.49	11922.55
12	05981.05	05837.84	04965.66
13	01396.19	01386.80	01228.66
14	33243.23	33762.48	31635.63
15	28530.54	27886.25	25300.40
16	07584.34	07164.82	06932.04
17	02030.83	01981.03	01655.80
18	00534.05	00534.05	00534.05

Problem #	Expected Total Inspection Cost (with material handling) Using:		
	Raouf et al Model	Model (1) in the thesis	Model (2) in the thesis
19	15934.30	15762.60	14265.87
20	08909.73	08648.14	08348.02
21	04562.60	04465.25	04465.25
22	01332.08	01326.32	01326.32
23	22010.86	21750.76	20690.75
24	23205.36	23346.82	18406.94
25	04872.50	04783.04	04783.04
26	02436.97	02418.10	02350.44
27	31449.68	31647.80	29408.52
28	10049.82	09917.81	09312.97
29	07541.17	07430.14	06411.92
30	05000.51	04767.32	04472.00
31	00759.85	00759.85	00759.85
32	24710.43	24624.32	21432.19
33	10512.75	09984.94	09620.52
34	02917.70	02892.72	02841.39
35	01219.28	01179.05	01179.05
36	36163.16	36052.47	29914.45
37	11547.61	11137.96	10500.70
38	05926.13	05412.87	05412.87
39	02144.97	02144.02	02119.33
40	39545.61	40052.49	38285.24
41	21019.30	21091.37	18311.51
42	05470.16	05423.23	05204.70
43	02815.73	02751.79	02705.97
44	00869.80	00869.80	00869.80
45	29418.30	29642.45	26521.16
46	09400.52	09133.79	07515.56
47	05188.08	05160.78	04591.85
48	01746.27	01759.13	01620.66
49	13643.74	13643.37	13410.84

Problem #	Expected Total Inspection Cost (with material handling) Using:		
	Raouf et al Model	Model (1) in the thesis	Model (2) in the thesis
50	17772.02	16211.75	15171.27
51	06773.61	06281.40	06281.40
52	01655.35	01596.72	01548.07
53	39638.57	36728.14	29325.91
54	21164.69	21303.75	20447.04
55	06133.11	05907.95	05866.50
56	03664.99	03492.22	03492.22
57	00676.73	00676.73	00676.73
58	27201.86	27689.47	26956.64
59	08179.09	07970.82	06843.83
60	03424.92	03325.01	03234.80
61	01327.64	01322.27	01322.27
62	35270.55	35111.43	26702.73
63	07089.41	07237.28	07237.28
64	05580.25	05378.91	04660.93
65	01603.40	01602.17	01602.17
66	38546.05	39129.85	38024.66
67	17019.01	15110.51	12754.30
68	07956.90	07987.95	07895.16
69	03676.91	03656.77	03642.54
70	00520.60	00520.60	00520.60
71	21257.46	21175.77	19759.52
72	10446.94	09956.09	09956.09
73	04203.93	04098.17	03665.38
74	02028.18	02039.08	01883.95
75	21431.75	21710.97	20622.99
76	11158.60	10912.59	08557.85
77	05444.55	05412.89	05171.63
78	01849.73	01793.19	01793.19
79	23871.96	23889.83	23450.93
80	16925.44	16634.42	16634.42

Problem #	Expected Total Inspection Cost (with material handling) Using:		
	Raouf et al Model	Model (1) in the thesis	Model (2) in the thesis
81	05096.15	04820.40	04681.50
82	01645.41	01634.81	01611.28
83	00643.75	00643.75	00643.75
84	17431.24	17223.31	16895.08
85	06468.67	06501.75	06101.59
86	03149.54	03115.17	03115.17
87	00484.25	00484.25	00484.25
88	43714.58	43804.84	39859.36
89	13290.13	13199.50	11567.60
90	06023.92	05622.96	05622.96
91	01185.45	01174.33	01109.69
92	20454.74	20090.40	18952.89
93	17993.15	17331.43	16011.35
94	05117.70	05279.05	04820.37
95	02926.85	02921.55	02689.26
96	00336.93	00336.93	00336.93
97	07718.63	07615.60	07404.17
98	08828.39	08471.12	08263.31
99	05714.87	05429.68	05135.06
100	00938.29	00938.29	00938.29

Vita

Hazem Jawad Al-Najjar was born in Taif, Saudi Arabia, on September 1, 1963. After graduating from Al-Taif Secondary School, he attended King Fahd University of Petroleum and Minerals at Dhahran, Saudi Arabia. He received the degree of Bachelor of Science with a major in Systems Engineering in January 1986. In 1987, he was employed in the office of the Registrar at the same university as a Scheduling Unit Coordinator. Meanwhile, he was doing his masters as a part-time student. He received the degree of Master of Science in Systems Engineering from the same university in 1993.